Write Down Your NAME

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Last First

Circle Your DIVISION

Div. 1	Div. 2	Div. 3	Div.4
8:30 am	**9:30 pm**	**12:30 pm**	**3:30 pm**
Han	**Xu**	**Ruan**	**Pan**

ME315 - Heat and Mass Transfer
School of Mechanical Engineering
Purdue University

Final Exam
December 13, 2017

Read Instructions Carefully:

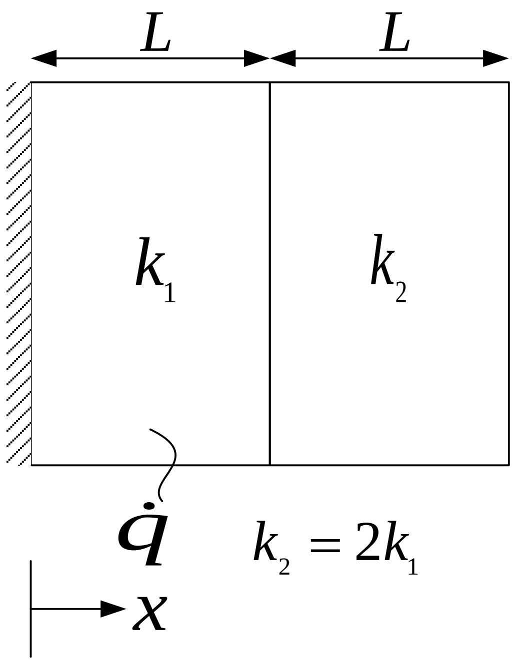
- Write your name on **each page** and circle your division number.
- Equation sheet and tables are attached to this exam. Three pages of US letter-size crib sheets are allowed.
- No books, notes, and other materials are allowed.
- ME Exam Calculator Policy is enforced. Only TI-30XIIS and TI-30XA are allowed.
- **Power off** all other digital devices, such as computer/tablet/phone and smart watch/glasses.
- Keep all the pages in order.
- You are asked to write your assumptions and answers to sub-problems in designated areas. Write on front side of the page only. If needed, you can insert extra pages but mark this clearly in the designated areas.

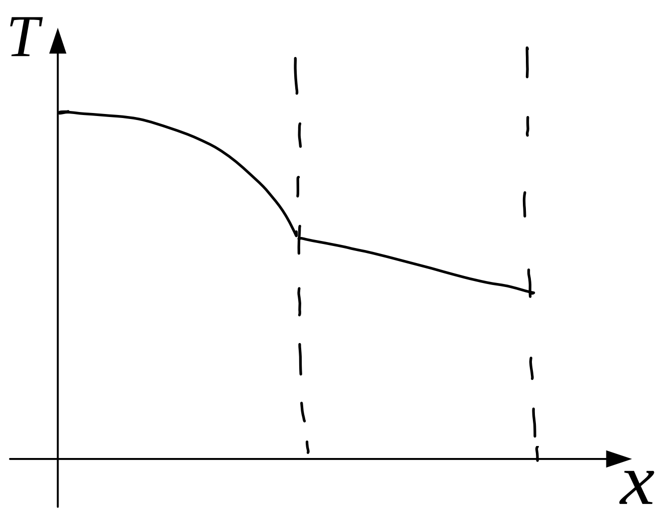
Performance		
1	**25**	
2	**25**	
3	**25**	
4	**25**	
Total	**100**	

Circle your division: 1 2 3 4 Name________________________________

Problem 1 [25 points]

(a) [6 points] Consider a system consisting of two solid slabs, each of thickness L. The first slab is insulated on the left side, and a uniform volumetric heat generation at a rate ($\dot{q}$) occurs in it. The thermal conductivity of the second slab is twice that of the thermal conductivity of the first slab. Assume one-dimensional heat conduction, steady-state conditions, and perfect contact between the two slabs. Sketch the temperature distribution through the two slabs, labeling correctly important slopes.





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Problem 1 – continued

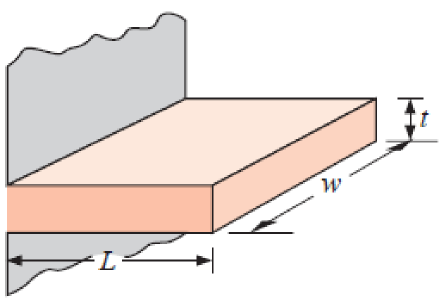
(b) [6 points] Two rectangular fins built from the same material are designed such that they have the same cross sectional area but different aspect ratio. Fin 1 has w_1=10 cm and t_1=2 mm, and Fin 2 has w_2=5 cm and t_2=4 mm. Both fins have the same length L and can be assumed as long fins. Apparently both fins use the same amount of materials to make, but which fin design is more effective in enhancing heat transfer? Justify your answer.

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q_f	
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\rvert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	(3.75)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$	(3.77)
B	Adiabatic: $d\theta/dx\rvert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	(3.80)	$M \tanh mL$	(3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	(3.82)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$	(3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	(3.84)	M	(3.85)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$



Fin 1 is more effective.

$q = \sqrt{hPkA_c}\,\theta_b$ or $\varepsilon_f = \sqrt{\frac{kP}{hA_c}}$

$P_1 = 2(t_1 + w_1) = 0.204\ m$

$P_2 = 2(t_2 + w_2) = 0.108\ m$

$P_1 > P_2$

Problem 1 – continued

(c) [7 points] A cup of hot coffee is initially at a uniform temperature of 80°C, while the ambient air is at a constant temperature of 25°C. Assume the paper cup to be a cylinder with diameter of 6 cm and height of 10cm, and neglect heat transfer from the top and bottom surfaces. The convective heat transfer coefficient *h* outside of the cup is 10W/m^2-K. The thermal conductivity of coffee can be assumed as that of water, 0.67W/m-K. The conduction resistance of the cup wall can be neglected.

Can you assume the temperature of the coffee to be uniform as it cools down? Justify your answer.

Solution:

$$L_c \equiv \frac{V}{A_s} = \frac{\frac{\pi D^2}{4} L}{\pi D L} = \frac{D}{4} = 0.015\text{m}$$

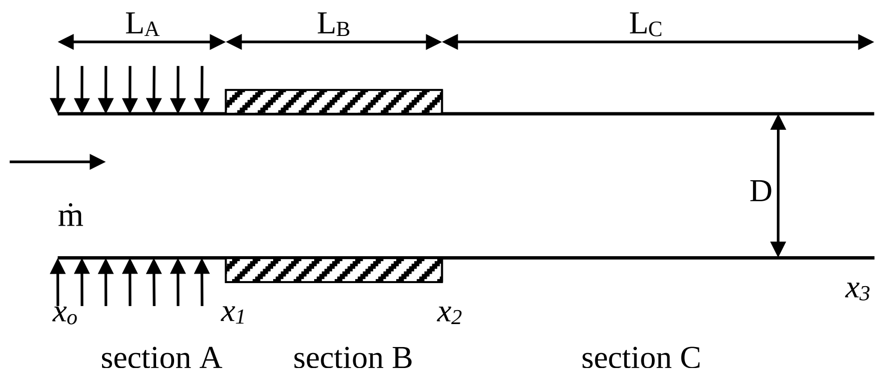
$$Bi = \frac{h L_c}{k} = \frac{10 \times 0.015}{0.67} = 0.22$$

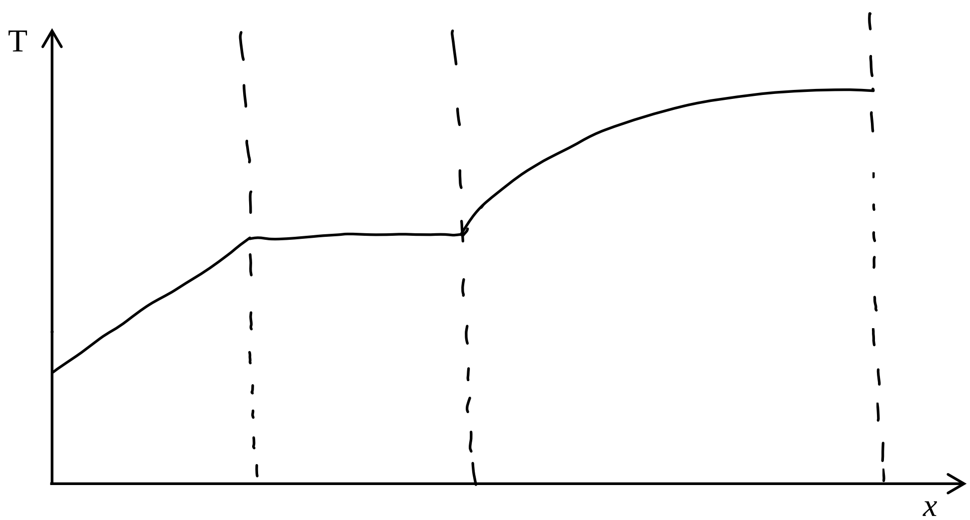
$\therefore$ Not uniform

Problem 1 – continued

(d) [6 points] Water is supplied at a flow rate $\dot{m}$ and temperature T_i to a circular tube of diameter *D*. Different boundary conditions are applied at different locations along the length of the tube:
Section A: uniform heat flux, q''_w
Section B: insulated
Section C: uniform wall temperature T_w with a value equal to the wall temperature at the end of Section A

Qualitatively sketch the mean water temperature along the tube. Describe the key features of your curves of each section.





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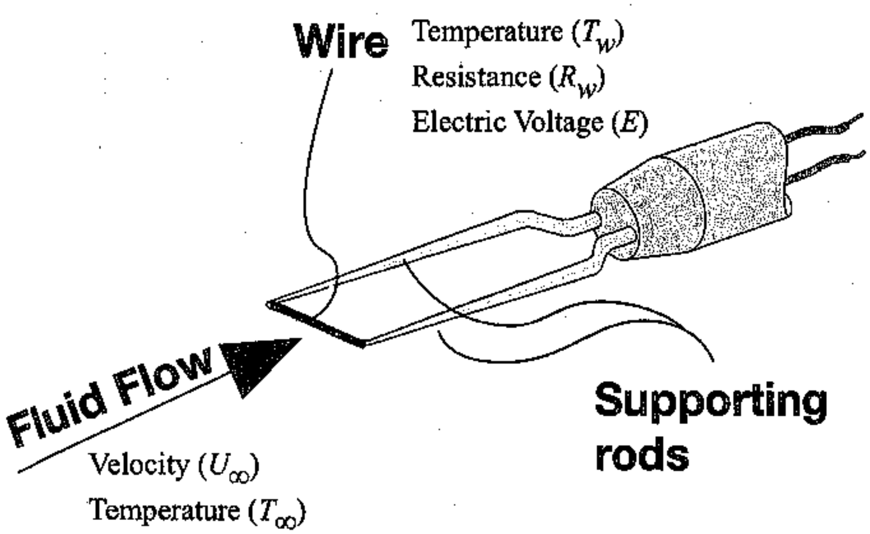
Problem 2 [25 points]

Consider a hot-wire anemometer as shown below. The anemometer is to measure the velocity of fluid flow by analyzing heat transfer around a wire whose diameter is D, length is L and temperature is maintained at a constant temperature (T_w). During operation, electric voltage (E) is applied to the wire, whose resistance is R_w, so that the heat is generated and dissipated to the fluid whose kinematic viscosity, thermal conductivity and diffusivity are ν, k_f, and α respectively. A feedback control circuit is employed to adjust the electric voltage to maintain the wire temperature at a given operating temperature. Design conditions pertain to an airstream at $T_\infty = 20°C$ and $1 \leq U_\infty \leq 50$ m/s with a wire temperature of $T_w = 100$ °C. Answer the following questions.



(a) Formulate the relation between the electric voltage (E) and the flow velocity (U_∞) by performing energy balance analysis around the wire using given symbols and relevant empirical correlations in the Equation sheet. You may neglect the conduction heat loss through the supporting rods.

(b) If the accuracy of the wire temperature control is ± 2.0 °C, what is the uncertainty of in the velocity measurements? If the wire temperature increases to 200 °C, what will be the uncertainty?

(c) If the conduction loss through the supporting rod becomes not negligible and reaches 10% of the heat generated along the wire, explain whether the value of measured velocity over-predict or under-predict the true flow velocity (U_∞). Your explanation should be based on the energy balance analysis around the wire.

(a) Energy Balance around the Wire

$$q_{conv} = h \cdot (\pi D \cdot L)(T_w - T_\infty)$$

$$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

$\dot{E}_{st} = 0$ ($\because$ steady state)

$\dot{E}_{in} = 0$ ($T_w > T_\infty$)

$\dot{E}_{out} = q_{conv} = h(\pi D \cdot L)(T_w - T_\infty)$

$\dot{E}_g = \dfrac{E^2}{R_w}$

$$\therefore \boxed{\frac{E^2}{R_w} = h(\pi D \cdot L)(T_w - T_\infty)}$$

From the Empirical Correlation for a Cylinder in Cross Flow,

$$\overline{Nu_D} = 0.683\, Re_D^{1/2} \cdot Pr^{1/3}$$

$$\frac{h \cdot D}{k_f} = 0.683 \left(\frac{U_\infty \cdot D}{\nu}\right)^{1/2} \left(\frac{\nu}{\alpha}\right)^{1/3}$$

$$\therefore h = \frac{0.683 \cdot k_f}{D^{1/2}\, \nu^{2/3} \cdot \alpha^{1/3}} \cdot U_\infty^{1/2}$$

$$\therefore \quad \boxed{\frac{E^2}{R_w} = \frac{0.683\, k_f \cdot \pi}{D^{1/2} \cdot \nu^{2/3} \cdot \alpha^{1/3}} (T_w - T_\infty) \cdot U_\infty^{1/2}}$$

→ Mistakes in computation

(b) Rearrange the solution of Part (a)
w.r.t. U_∞

$$U_\infty = \left[\left(\frac{E^2}{R_w} \right) \left(\frac{D^{1/2} \cdot \nu^{2/3} \cdot \alpha^{1/3}}{0.683\, k_f \pi} \right) \frac{1}{T_w - T_\infty} \right]^2$$

→ Uncertainty quantification

$$\left(\frac{\delta U_\infty}{U_\infty} \right)^2 = \left(2 \cdot \frac{\delta T_w}{T_w - T_\infty} \right)^2 \quad \text{or} \quad \left| \frac{\delta U_\infty}{U_\infty} \right| = \left| 2 \frac{\delta T_w}{T_w - T_\infty} \right|$$

$$\boxed{\frac{\delta U_\infty}{U_\infty} = \frac{2 \times 2.0°C}{80°C} = \pm 5\%} \quad \text{or} \quad \boxed{\delta U_\infty = 0.05\, U_\infty}$$

→ $T_w = 200°C$, $\boxed{\frac{\delta U_\infty}{U_\infty} = 2.2\% \text{ or } \delta U_\infty = 0.022\, U_\infty}$

(c) If the conduction loss is 10% of heat generation,

~~Ein~~ $E_{out} = q_{conv} + q_{conv}$.

$$= h \cdot (\pi D L)(T_w - T_\infty) + (0.1)\frac{E^2}{R_w}$$

→ Overall energy Balance

$$\frac{E^2}{R_w} = h \cdot (\pi D L)(T_w - T_\infty) + 0.1 \cdot \frac{E^2}{R_w}.$$

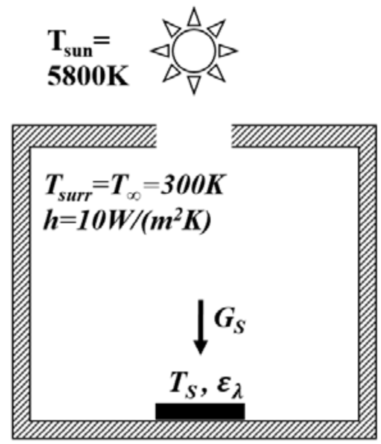
$$\boxed{0.9\frac{E^2}{R_w} = h(\pi D L)(T_w - T_\infty)}$$

For a given voltage reading (i.e., E = const), lower ~~higher~~ value of U_∞ is required for energy balance. Thus, the velocity reading will ~~under~~ **over-predict** the real flow velocity.

Problem 3 [25 points]

Inside a large room, a small thin flat plate is fully coated with an opaque layer of paint, horizontally placed under direct sun light through a small open window. The backside of the plate is thermally insulated. The sun is known to have a surface temperature of T_{sun}=5800K and the solar irradiation onto the plate surface is known as G_S=500W/m^2. The ambient air temperature is T_∞=300K with a natural convective heat transfer coefficient of h=10W/(m^{2}K). The room wall also has a temperature of T_{surr}=300K and an emissivity of ε_{surr}=0.5. At a certain time instant, the plate has at a uniform temperature of T_S=320K. The paint surface is diffuse and its spectral emissivity is known to be:

$$\varepsilon_\lambda = \begin{cases} 0.2 & for\ \ 0 \le \lambda \le 1\ \mu m \\ 0.8 & for\ \ \lambda > 1\ \mu m \end{cases}.$$



(a) Calculate the total surface emissivity (ε) of the plate.
(b) Calculate the total absorptivity of the plate to solar radiation (α_S) and the total absorptivity of the plate to radiation from the wall of the room (α_{surr}).
(c) Calculate the total irradiation (G) onto the plate and the total radiosity (J) of the plate (W/m^2).
(d) Calculate the net heat flux of the plate at this moment (W/m^2).

Blackbody Radiation Functions (Table 12.2 from Textbook)

λT ($\mu m \cdot K$)	$F_{(0\to\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\dfrac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.696720
4,600	0.579280	0.467343	0.647004
4,800	0.607559	0.433109	0.599610
5,000	0.633747	0.400813	0.554898
5,200	0.658970	0.370580×10^{-4}	0.513043
5,400	0.680360	0.342445	0.474092
5,600	0.701046	0.316376	0.438002
5,800	0.720158	0.292301	0.404671
6,000	0.737818	0.270121	0.373965
6,200	0.754140	0.249723×10^{-4}	0.345724
6,400	0.769234	0.230985	0.319783
6,600	0.783199	0.213786	0.295973
6,800	0.796129	0.198008	0.274128
7,000	0.808109	0.183534	0.254090
7,200	0.819217	0.170256×10^{-4}	0.235708
7,400	0.829527	0.158073	0.218842
7,600	0.839102	0.146891	0.203360
7,800	0.848005	0.136621	0.189143
8,000	0.856288	0.127185	0.176079
8,500	0.874608	0.106772×10^{-4}	0.147819
9,000	0.890029	0.901463×10^{-5}	0.124801
9,500	0.903085	0.765338	0.105956
10,000	0.914199	0.653279×10^{-5}	0.090442
10,500	0.923710	0.560522	0.077600
11,000	0.931890	0.483321	0.066913
11,500	0.939959	0.418725	0.057970
12,000	0.945098	0.364394×10^{-5}	0.050448
13,000	0.955139	0.279457	0.038689
14,000	0.962898	0.217641	0.030131
15,000	0.969981	0.171866×10^{-5}	0.023794
16,000	0.973814	0.137429	0.019026
18,000	0.980860	0.908240×10^{-6}	0.012574
20,000	0.985602	0.623310	0.008629
25,000	0.992215	0.276474	0.003828
30,000	0.995340	0.140469×10^{-6}	0.001945
40,000	0.997967	0.473891×10^{-7}	0.000656
50,000	0.998953	0.201605	0.000279
75,000	0.999713	0.418597×10^{-8}	0.000058
100,000	0.999905	0.135752	0.000019

Problem 3 – continued

List your assumptions below. [3 pts]

No conductions
Very large room with isothermal surface temperature
Constant parameters
Uniform irradiation and radiosity for the plate surface
Uniform plate temperature, etc.

(Note: It is not steady state. Not grey surfaces.)

Start you answer to part (a) here. [5 pts]

$$\varepsilon_T = \varepsilon_{(0-1um)} F_{(0-1um)@320K} + \varepsilon_{(1um-\infty)} (1 - F_{(0-1um)@320K}) = 0.2 \times 0 + 0.8 \times 1 = 0.8$$

Start you answer to part (b) here. [6 pts]

$$\alpha_S = \alpha_{(0-1um)} F_{(0-1um)@5800K} + \alpha_{(1um-\infty)} (1 - F_{(0-1um)@5800K}) = 0.2 \times 0.72 + 0.8 \times 0.28 = 0.368 \;;$$

$$\alpha_{surr} = \alpha_{(0-1um)} F_{(0-1um)@300K} + \alpha_{(1um-\infty)} (1 - F_{(0-1um)@300K}) = 0.2 \times 0 + 0.8 \times 1 = 0.8 \;;$$

Circle your division: 1 2 3 4 Name_________________________________

Problem 3 – continued

Start your answer to part 3(c) here: [6 pts]

$$G = G_S + G_{Surr} = 500 + E_b(300K) = 959\left[\frac{W}{m^2}\right];$$

$$\begin{aligned} J &= E + \rho_S G_S + \rho_{Surr} G_{Surr} = \varepsilon_T E_b(T_S) + (1-\varepsilon_S)G_S + (1-\varepsilon_{Surr})G_{Surr} \\ &= 0.8 \times E_b(320K) + (1-0.368)\times 500 + (1-0.8)\times E_b(300K) \\ &= 883\left[\frac{W}{m^2}\right] \end{aligned}$$

Start your answer to part 3(d) here: [5 pts]

The net heat flux out of the plate:

$$q''_{net} = (J - G) + h(T_S - T_\infty) = (883 - 959) + 10\times(320-300) = 124\left[\frac{W}{m^2}\right].$$

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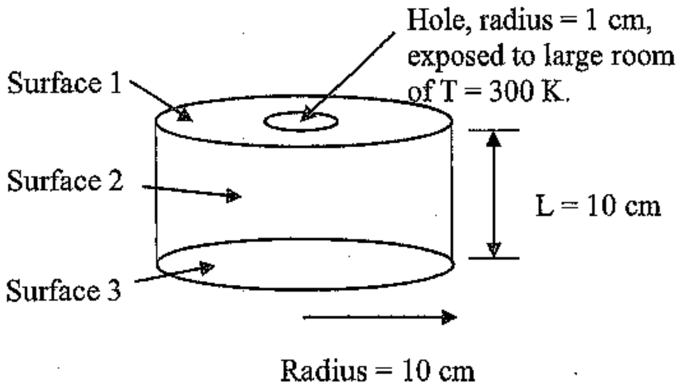
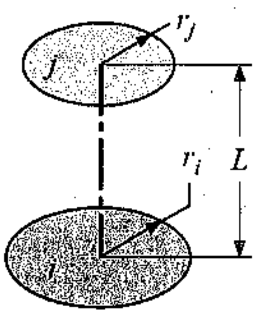
Problem 4 [25 points]

A cylindrical furnace has dimensions as shown in the figure below. There is a hole on the top surface of the furnace, whose radius r_h is 1 cm. Surface 1 (the top surface excluding the hole) and surface 3 (the bottom surface) are black, and surface 2 (the side wall) is diffuse gray with an emissivity of 0.8. The furnace is exposed to a large room whose wall temperature is 300 K. All the surfaces of the furnace (surface 1, 2, and 3) are maintained at a steady state temperature of 500 K.



(a) Calculate the view factors F_{21}, F_{23}, and F_{24}. A formula of view factor is given below.

Coaxial Parallel Disks (Figure 13.5)



$$R_i = r_i/L,\ R_j = r_j/L$$

$$S = 1 + \frac{1 + R_j^2}{R_i^2}$$

$$F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$$

(b) For surface 1, 2, and 3 of the furnace, and the hole (call it surface 4), write the equation of radiosity, i.e., the radiation equation relating surface temperature or radiation heat flux with radiosity.
(c) Calculate the radiosity of each surface, J_1, J_2, J_3, and J_4 (W/m^2).
(d) Calculate the radiative power (W) emitted from the furnace through the hole.

Circle your division: 1 2 3 4 Name________________________________

Problem 4 – continued

List your assumptions below [2 points]

Openning can be approximated as a black body @ 300K
Radiosity is uniform over each surface
steady state, gray surface for 2, black body for 1, 3.

Start your answer to part (a) here. [6 points]

$F_{3(1+4)}$ = (from formula) = 0.382

$F_{32} = 1 - F_{3(1+4)}$ $\quad F_{23} = \frac{A_3 F_{32}}{A_2} = \boxed{0.309}$

F_{34} from formula = 0.005

Using reciprocity, summation rules: $F_{24} = \boxed{0.0025}$

$F_{21} = F_{2(1+4)} - F_{24} = \boxed{0.306}$

Start your answer to part (b) here. [4 points]

① : $J_1 = \sigma T_1^4$

② $\frac{E_{b2} - J_2}{\frac{1-\varepsilon_2}{\varepsilon_2 A_2}} = \frac{J_2 - J_1}{\frac{1}{A_2 F_{21}}} + \frac{J_2 - J_3}{\frac{1}{A_2 F_{23}}} + \frac{J_2 - J_4}{\frac{1}{A_2 F_{24}}}$

③ $J_3 = \sigma T_3^4$

④ $J_4 = \sigma T_4^4$ $\quad T_4 = 300K$

Circle your division: 1 2 3 4 Name________________________

Problem 4 – continued

Start your answer to part (c) here. [8 points]

$J_1 = 3544\ W/m^2$

$J_2 = 3542\ W/m^2$

$J_3 = 3544\ W/m^2$

$J_4 = 459\ W/m^2$

Start your answer to part (d) here. [5 points]

$q_{to\,4} = q_{2\to 4} + q_{3\to 4} = 1.1\ W$