ME 315
Exam 3
8:00 -9:00 PM
Thursday, April 16, 2009

- This is a closed-book, closed-notes examination. *There is a formula sheet at the back.*
- You must turn off all communications devices before starting this exam, and leave them off for the entire exam.
- Please write legibly and show all work for your own benefit. Please show your final answers in the boxes provided.
- State all assumptions.
- Please arrange all your sheets in the correct order. Make sure they are all included.

Name: ___________________________________

Last   First

CIRCLE YOUR DIVISION

Div. 1 (9:30 am)
Prof. Murthy

Div. 2 (12:30 pm)
Prof. Choi

Problem	Score
1 (30 Points)	
2 (30 Points)	
3 (40 Points)	
Total (100 Points)	

1. (30 points) Consider a heat exchanger with a heat exchange area of 30 m^2 and operating under the conditions listed in the table below.

	Hot Fluid	Cold Fluid
Heat Capacity C (kW/K)	5	2
Inlet temperature ^oC	60	45
Outlet temperature ^oC	--	57.5

(i) Determine the outlet temperature of the hot stream in ^oC.

T_{ho} =

(ii) Is the heat exchanger operating in

a. Parallel flow
b. Counterflow
c. Can't tell

Circle only one answer and explain your choice.

Explanation:

(iii) Calculate the overall heat transfer coefficient U (W/m^2K).

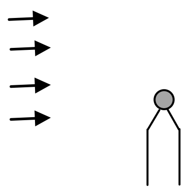
U =

(iv) What would the effectiveness be if the heat exchanger were infinitely long?

ε =

2. (30 points) You are measuring the combustion gas temperature at the turbine inlet in a gas turbine engine using a thermocouple junction. The thermocouple has a diameter of 1 mm, and is initially at T_i = 25 °C. You may neglect radiation in the following analysis.

Hot combustion gas
(evaluated at the
free stream temperature)
k = 0.04 W/mK
ρ = 0.6 kg/m^3
c_p = 1000 J/kgK
ν = 50 x 10^{-6} m^2/s
U = 20 m/s



Thermocouple
k = 300 W/mK
ρ = 9000 kg/m^3
c_p = 400 J/kgK

μ_s = 25 x 10^{-6} kg/ms (gas viscosity evaluated at the thermocouple surface temperature, T_s)

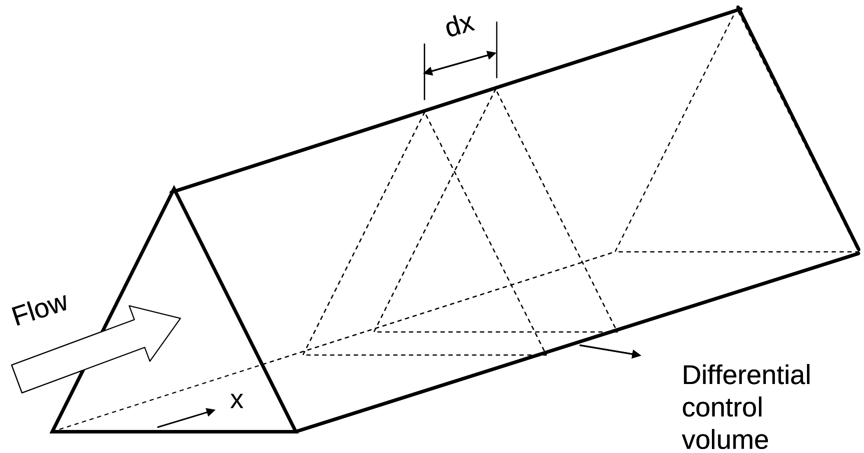
(i) Determine the average convective heat transfer coefficient, $\bar{h}$.

$\bar{h}$ = W/m^{2}K

(ii) The temperature measured at the thermocouple centerline after 3 sec is 900 °C. Determine the combustion gas temperature, T_∞.

T_∞ = K

3. (40 points) Fluid enters a duct of triangular cross-section with a mass flow rate of $\dot{m}$. The cross-section is an equilateral triangle of side s. The fluid is heated at the walls with a heat flux $q'' = ax + b$ (W/m^2) into the domain, where x is measured from the entrance to the duct. Furthermore, a chemical reaction in the volume of the fluid causes a constant volumetric heat generation rate of $\dot{q}$ W/m^3. The inlet bulk temperature of the fluid is T_{mi}. You are given that the fluid has a constant density ρ and a constant specific heat C_p.



(a) By considering the differential control volume shown in the figure, write an energy balance to derive a <u>symbolic expression</u> for $\frac{dT_m}{dx}$ as a function of a, b, s, x, $\dot{m}$, $\dot{q}$ and the physical properties of the fluid.

$$\frac{dT_m}{dx} =$$

(b) Derive a <u>symbolic expression</u> for the variation of T_m with x in terms of a, b, s, x, $\dot{m}$, $\dot{q}$, the inlet bulk temperature T_{mi} and the physical properties of the fluid.

$T_m(x) =$

(c) Is the expression you derived in part (b) for $T_m(x)$

(i) always valid for this problem?

(ii) only valid for this problem if the flow is fully developed but the temperature is developing?

(iii) only valid for this problem if the flow and temperature are fully-developed?

Circle only one answer, and justify your choice.

FORMULA SHEET

Conservation Laws

Control Volume Energy Balance: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$; $\dot{E}_{st} = mC_p \, dT/dt$; $\dot{E}_{gen} = \dot{q}V$

Surface Energy Balance: $\dot{E}_{in} - \dot{E}_{out} = 0$

Conduction

Fourier's Law: $q''_{cond,x} = -k\dfrac{\partial T}{\partial x}$; $q''_{cond,n} = -k\dfrac{\partial T}{\partial n}$; $q_{cond} = q''_{cond}A$

Heat Flux Vector: $\vec{q''} = q''_x\vec{i} + q''_y\vec{j} + q''_z\vec{k} = -k\left[\dfrac{\partial T}{\partial x}\vec{i} + \dfrac{\partial T}{\partial y}\vec{j} + \dfrac{\partial T}{\partial z}\vec{k}\right]$

Heat Diffusion Equation:

Rectangular Coordinates: $\dfrac{\partial}{\partial x}\left(k\dfrac{\partial T}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(k\dfrac{\partial T}{\partial y}\right) + \dfrac{\partial}{\partial z}\left(k\dfrac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p\dfrac{\partial T}{\partial t}$

Cylindrical Coordinates: $\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(kr\dfrac{\partial T}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial}{\partial \phi}\left(k\dfrac{\partial T}{\partial \phi}\right) + \dfrac{\partial}{\partial z}\left(k\dfrac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p\dfrac{\partial T}{\partial t}$

Spherical Coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho C_p\frac{\partial T}{\partial t}$$

Thermal Resistance Concepts:

Conduction Resistance: $R_{t,cond} \overset{plane\,wall}{=} \dfrac{L}{kA}$; $R_{t,cond} \overset{cylinder}{=} \dfrac{\ln(r_o/r_i)}{2\pi lk}$; $R_{t,cond} \overset{sphere}{=} \dfrac{(1/r_i)-(1/r_o)}{4\pi k}$

Convection Resistance: $R_{t,conv} \overset{plane\,wall}{=} \dfrac{1}{h_{conv}A}$; $R_{t,conv} \overset{cylinder}{=} \dfrac{1}{2\pi rlh_{conv}}$; $R_{t,conv} \overset{sphere}{=} \dfrac{1}{4\pi r^2h_{conv}}$

Radiation Resistance: $R_{t,rad} \overset{plane\,wall}{=} \dfrac{1}{h_{rad}A}$; $R_{t,rad} \overset{cylinder}{=} \dfrac{1}{2\pi rlh_{rad}}$; $R_{t,rad} \overset{sphere}{=} \dfrac{1}{4\pi r^2h_{rad}}$

Combined Convection and Radiation Surface: $\dfrac{1}{R_{conv+rad}} = \dfrac{1}{R_{t,conv}} + \dfrac{1}{R_{t,rad}}$

Contact Resistance: $R_{t,contact} = \dfrac{1}{h_{contact}A_{contact}}$

Thermal Energy Generation:

$T(x) - T_s \overset{plane\ wall}{=} \dfrac{\dot{q}L^2}{2k}\left(1 - \dfrac{x^2}{L^2}\right)$; $\quad T(r) - T_s \overset{cylinder}{=} \dfrac{\dot{q}r_o^2}{4k}\left(1 - \dfrac{r^2}{r_o^2}\right)$

Extended Surfaces:

Convective Tip: $\dfrac{\theta(x)}{\theta_b}=\dfrac{\cosh\left[m(L-x)\right]+(h/mk)\sinh\left[m(L-x)\right]}{\cosh(mL)+(h/mk)\sinh(mL)}$

$$q_{fin}=\left(hPkA_c\right)^{1/2}\theta_b\frac{\sinh(mL)+(h/mk)\cosh(mL)}{\cosh(mL)+(h/mk)\sinh(mL)}$$

Adiabatic Tip: $\dfrac{\theta(x)}{\theta_b}=\dfrac{\cosh\left[m(L-x)\right]}{\cosh(mL)}$; $q_{fin}=\left(hPkA_c\right)^{1/2}\theta_b\tanh(mL)$

Prescribed Tip Temperature: $\dfrac{\theta(x)}{\theta_b}=\dfrac{(\theta_L/\theta_b)\sinh(mx)+\sinh\left[m(L-x)\right]}{\sinh(mL)}$

$$q_{fin}=\left(hPkA_c\right)^{1/2}\theta_b\frac{\cosh(mL)-(\theta_L/\theta_b)}{\sinh(mL)}$$

Infinitely Long Fin: $\dfrac{\theta(x)}{\theta_b}=e^{-mx}$; $q_{fin}=\left(hPkA_c\right)^{1/2}\theta_b$

$$m^2=\frac{hP}{kA_c};\ \theta_b=T_b-T_\infty\ ;\ q_{fin}=q_{conv,\,finsurface}+q_{conv,tip};\ q_{conv,tip}=hA_c\theta_L$$

Fin Effectiveness: $\varepsilon_{fin}=\dfrac{q_{fin}}{hA_{c,b}\theta_b}$; $\varepsilon_{fin}=\dfrac{R_{t,conv-base}}{R_{t,cond-fin}}$

Fin Efficiency: $\eta_{fin}=\dfrac{q_{fin}}{hA_{fin}\theta_b}$; $\eta_{fin}\overset{adiabatic}{=}\dfrac{\tanh(mL)}{mL}$; $L_c=L+\dfrac{A_c}{P}$; $\eta_{fin}=\dfrac{\tanh(mL_c)}{mL_c}$

$$\eta_o=\frac{q_{total}}{hA_{total}\theta_b}=1-\frac{NA_{fin}}{A_{total}}\left(1-\eta_{fin}\right);\ R_{t,cond-fin}=\frac{1}{\eta_{fin}hA_{fin}};\ R_{t,cond-fin\,array}=\frac{1}{\eta_o hA_{total}}$$

Two Dimensional Steady Conduction:

Conduction Shape Factor: $R_{t,cond}\overset{2D}{=}\dfrac{1}{Sk}$

Finite Difference Method: $T_{i+1,j}+T_{i-1,j}+T_{i,j+1}+T_{i,j-1}=4T_{i,j}$

Transient Conduction:

Lumped System Analysis: $Bi=\dfrac{R_{t-cond}}{R_{t-conv}}=\dfrac{h_{conv}L_c}{k_{solid}}$; $\dfrac{\theta}{\theta_i}=\dfrac{T-T_\infty}{T_i-T_\infty}=\exp\left(-\dfrac{t}{\tau_t}\right)$; $Fo=\dfrac{\alpha t}{L_c^2}$

$$\frac{\theta}{\theta_i}=\exp\left[-\left(\frac{h_{conv}L_c}{k_{solid}}\right)\left(\frac{\alpha t}{L_c^2}\right)\right]=\exp\left[-(Bi)(Fo)\right];\ \tau_t=\frac{\rho VC_p}{h_{conv}A_s}=C_{t,solid}R_{t,conv};$$

Analytical Solutions: $\theta^*=\dfrac{\theta}{\theta_i}=\dfrac{T-T_\infty}{T_i-T_\infty}$; $x^*=\dfrac{x}{L}$; $r^*=\dfrac{r}{r_o}$; $t^*=\dfrac{\alpha t}{L^2}$

Plane Wall: $\theta^*\underset{plane\,wall}{\cong}C_1\exp\left(-\zeta_1^2Fo\right)\cos\left(\zeta_1x^*\right)$; $\theta_o^*\overset{plane\,wall}{=}C_1\exp\left(-\zeta_1^2Fo\right)$; $\dfrac{Q}{Q_o}\overset{plane\,wall}{=}1-\theta_o^*\dfrac{\sin(\zeta_1)}{\zeta_1}$

Long Cylinder: $\theta^*\underset{cylinder}{\cong}C_1\exp\left(-\zeta_1^2Fo\right)J_0\left(\zeta_1r^*\right)$; $\theta_o^*\overset{cylinder}{=}C_1\exp\left(-\zeta_1^2Fo\right)$; $\dfrac{Q}{Q_o}\overset{cylinder}{=}1-2\theta_o^*\dfrac{J_1(\zeta_1)}{\zeta_1}$

Sphere: $\theta^* \underset{sphere}{\cong} C_1 \exp\left(-\zeta_1^2 Fo\right) \frac{\sin\left(\zeta_1 r^*\right)}{\zeta_1 r^*}$; $\theta_o^* \overset{sphere}{=} C_1 \exp\left(-\zeta_1^2 Fo\right)$;

$\frac{Q}{Q_o} \overset{sphere}{=} 1 - 3\theta_o^* \frac{\left[\sin\left(\zeta_1\right) - \zeta_1 \cos\left(\zeta_1\right)\right]}{\zeta_1^3}$

Semi-infinite Solid:

Constant Surface Temperature: $\frac{T\left(x,t\right) - T_s}{T_i - T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$; $q_s'' = -k \left.\frac{\partial T}{\partial x}\right|_{x=0} = \frac{k\left(T_s - T_i\right)}{\sqrt{\pi \alpha t}}$

Constant Surface Heat Flux: $T\left(x,t\right) - T_i = \frac{2q_0''\left(\alpha t/\pi\right)^{1/2}}{k} \exp\left(-\frac{x^2}{4\alpha t}\right) - \frac{q_0'' x}{k} erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$

Convection: $\frac{T\left(x,t\right) - T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\right]\left[erfc\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$

Finite Difference Method:

Explicit Method: $T_{i,j}^{P+1} \overset{explicit}{=} \left(1 - 4Fo\right) T_{i,j}^P + Fo\left(T_{i+1,j}^P + T_{i-1,j}^P + T_{i,j+1}^P + T_{i,j-1}^P\right)$

Implicit Method: $T_{i,j}^{P} \overset{implicit}{=} \left(1 + 4Fo\right) T_{i,j}^{P+1} - Fo\left(T_{i+1,j}^{P+1} + T_{i-1,j}^{P+1} + T_{i,j+1}^{P+1} + T_{i,j-1}^{P+1}\right)$

Stability Limits: $\Delta t \underset{explicit}{\overset{1D}{\le}} \frac{\left(\Delta x\right)^2}{2\alpha}$; $\Delta t \underset{explicit}{\overset{2D}{\le}} \frac{\left(\Delta x\right)^2}{4\alpha}$; $\Delta t \underset{explicit}{\overset{3D}{\le}} \frac{\left(\Delta x\right)^2}{6\alpha}$

Convection

Newton's Law of Cooling: $q_{conv}'' = h_{conv}\left(T_s - T_\infty\right)$; $q_{conv} = q_{conv}'' A$

Mass Transfer: $n_A'' = h_m\left(\rho_{A,s} - \rho_{A,\infty}\right)$; $q_{evap} = n_A'' A h_{fg}$

Average Heat Transfer Coefficient: $\overline{h_{conv}} = \frac{1}{A_s} \int_{A_S} h_{conv} dA_s$

Average Mass Transfer Coefficient: $\overline{h_m} = \frac{1}{A_s} \int_{A_S} h_m dA_s$

Dimensionless Parameters:

Reynolds Number: $Re_{L_c} = \frac{\rho V L_c}{\mu} = \frac{V L_c}{\nu}$; Grashof Number: $Gr_{L_c} = \frac{g\beta\left(T_s - T_\infty\right) L_c^3}{\nu^2}$

Prandtl Number: $Pr = \frac{\nu}{\alpha}$; Schmidt Number: $Sc = \frac{\nu}{D_{AB}}$; Lewis Number: $Le = \frac{\alpha}{D_{AB}}$

Nusselt Number: $Nu = \frac{h_{conv} L_c}{k_{fluid}}$; Sherwood Number: $Sh = \frac{h_m L_c}{D_{AB}}$

Rayleigh Number: $Ra_{L_c} = Gr_{L_c} Pr = \frac{g\beta\left(T_s - T_\infty\right) L_c^3}{\nu^2} \frac{\nu}{\alpha} = \frac{g\beta\left(T_s - T_\infty\right) L_c^3}{\nu \alpha}$

Boundary Layer Thickness: $\frac{\delta}{\delta_t} \approx Pr^n$; $\frac{\delta}{\delta_c} \approx Sc^n$; $\frac{\delta_t}{\delta_c} \approx Le^n$

Heat-Mass Analogy: $\frac{Nu}{Sh} = \frac{Pr^n}{Sc^n}$; $\frac{h}{h_m} = \frac{k}{D_{AB} Le^n} = \rho C_p Le^{1-n}$

External Flow:

Flat Plate

Flat Plate (Laminar Local): $\delta \overset{flat\ plate}{\underset{laminar}{=}} \frac{5x}{Re_x^{1/2}}$; $C_{f,x} \overset{flat\ plate}{\underset{laminar}{=}} 0.664 Re_x^{-1/2}$;

$Nu_x \overset{isothermal\ flat\ plate}{\underset{laminar}{=}} 0.332 Re_x^{1/2} Pr^{1/3}$

Flat Plate (Laminar Average): $\overline{C_{f,L}} \overset{flat\ plate}{\underset{laminar}{=}} 1.328 Re_L^{-1/2}$; $\overline{Nu_L} \overset{isothermal\ flat\ plate}{\underset{laminar}{=}} 0.664 Re_L^{1/2} Pr^{1/3}$;

$\overline{Sh_L} \overset{flat\ plate}{\underset{laminar}{=}} 0.664 Re_L^{1/2} Sc^{1/3}$

Flat Plate (Turbulent Local): $\delta \overset{flat\ plate}{\underset{turbulent}{=}} \frac{0.37x}{Re_x^{1/5}}$; $C_{f,x} \overset{flat\ plate}{\underset{turbulent}{=}} 0.0592 Re_x^{-1/5}$;

$Nu_x \overset{isothermal\ flat\ plate}{\underset{turbulent}{=}} 0.0296 Re_x^{4/5} Pr^{1/3}$;

Flat Plate (Turbulent Average): $\overline{C_{f,L}} \overset{flat\ plate}{\underset{turbulent}{=}} 0.074 Re_L^{-1/5}$;

$\overline{Nu_L} \overset{isothermal\ flat\ plate}{\underset{turbulent}{=}} 0.037 Re_L^{4/5} Pr^{1/3}$

Flat Plate (Mixed Average): $\overline{C_{f,L}} \overset{flat\ plate}{\underset{mixed}{=}} 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$;

$\overline{Nu_L} \overset{isothermal\ flat\ plate}{\underset{mixed}{=}} \left(0.037 Re_L^{4/5} - 871\right) Pr^{1/3}$

Cylinder:

Cylinder: $\overline{Nu_D} \overset{cylinder}{=} 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1 + \left(0.4/Pr\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5}$ for $Re_D Pr > 0.2$;

Sphere:

$$\bar{N}u_D = 2 + \left(0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3}\right) \mathrm{Pr}^{0.4} \left(\frac{\mu}{\mu_s}\right)^{1/4}$$

Internal Flow:

Mean Velocity: $u_m = \frac{\int_{A_c} \rho u(r,x)\, dA_c}{\rho A_c}$; $u_m \overset{circular}{=} \frac{2}{r_o^2} \int_0^{r_o} u(r,x)\, r dr$

Reynolds Number: $Re_{D_h} = \frac{u_m D_h}{\nu}$; $D_h = \frac{4A_c}{P}$; $Re_D \overset{circular}{=} \frac{u_m D}{\nu}$

Turbulent: $Re_D \geq 2{,}300$

Hydrodynamic Entrance Lengths: $\left(\frac{x_{fd,hydrodynamic}}{D}\right)^{laminar} = 0.05 Re_D$; $60 > \left(\frac{x_{fd,hydrodynamic}}{D}\right)^{turbulent} > 10$

Thermal Entrance Lengths: $\left(\frac{x_{fd,thermal}}{D}\right)^{laminar} = 0.05 Re_D Pr$; $60 > \left(\frac{x_{fd,thermal}}{D}\right)^{turbulent} > 10$

Mean (Bulk) Temperature: $T_m = \frac{\int_{A_c} \rho u C_p T dA_c}{\dot{m} C_p}$; $T_m \overset{circular}{=} \frac{2}{u_m r_o^2} \int_0^{r_o} u T(r,x) r dr$

Constant Heat Flux: $T_m(x) = T_{m,i} + \frac{q''_{conv} P}{\dot{m} C_p} x = T_{m,i} + \frac{q''_s P}{\dot{m} C_p} x$; $q_{conv} = q''_s A_s = q''_s (PL)$

Constant Surface Temperature: $\frac{T_s - T_{m,x}}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m} C_p} \overline{h_{conv}}\right)$; $\Delta T_{LMTD} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m} C_p} \overline{h_{conv}}\right); \; q_{conv} = \overline{h_{conv}} A_s \Delta T_{LMTD} = \dot{m} C_p \left(T_{m,o} - T_{m,i}\right)$$

Circular Pipe

Fanning Friction Factor :

$f^{laminar} = \frac{64}{Re_D}$; $f \overset{turbulent}{=} \frac{0.316}{Re_D^{1/4}}$ for $Re_D < 2x10^4$; $f \overset{turbulent}{=} \frac{0.184}{Re_D^{1/5}}$ for $Re_D > 2x10^4$

Laminar Fully-developed Region: $Nu_D \underset{q''=constant}{\overset{laminar}{=}} 4.36$; $Nu_D \underset{T_s=constant}{\overset{laminar}{=}} 3.66$

Laminar Entrance Region: $\overline{Nu_D} \overset{T_s=constant}{=} 3.66 + \frac{0.0668 (D/L) Re_D Pr}{1 + 0.04 \left[(D/L) Re_D Pr\right]^{2/3}}$;

$\overline{Nu_D} \overset{T_s=constant}{=} 1.86 \left(\frac{Re_D Pr}{L/D}\right)^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$ (Both applicable to q_s'' = constant)

Turbulent Fully-Developed Region: $Nu_D \overset{turbulent}{=} 0.023 Re_D^{4/5} Pr^n$; $n \overset{fluid\ cooling}{=} 0.3$

$n \overset{fluid\ heating}{=} 0.4$ (Applicable to q_s'' = constant and T_s = constant)

Free Convection:

Boundary Layer Parameters: $\eta = \frac{y}{x} \left(\frac{Gr_x}{4}\right)^{1/4}$; $u = \frac{2\nu}{x} (Gr_x)^{1/2} f'(\eta)$

$Gr_x = \frac{g\beta (T_s - T_\infty) x^3}{\nu^2}$; $Ra_x = Gr_x Pr = \frac{g\beta (T_s - T_\infty) x^3}{\nu\alpha}$

Vertical Flat Plate: $\overline{Nu_L} \overset{isothermal\ vertical\ plate}{=} \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$

Horizontal Flat Plate: $L_c = A_s / P$; $\overline{Nu_{L_c}} \underset{upper\,hot/lower\,cold}{\overset{isothermal\ horizontal\ plate}{=}} = 0.54 Ra_{L_c}^{1/4}$;

$\overline{Nu_{L_c}} \underset{upper\,hot/lower\,cold}{\overset{isothermal\ horizontal\ plate}{=}} = 0.15 Ra_{L_c}^{1/3}$; $\overline{Nu_{L_c}} \underset{lower\,hot/upper\,cold}{\overset{isothermal\ horizontal\ plate}{=}} = 0.27 Ra_{L_c}^{1/4}$

Horizontal Cylinder: $\overline{Nu_D} \overset{isothermal\ horizontal\ cylinder}{=} C Ra_D^n$

Sphere: $\overline{Nu_D} \overset{isothermal\ sphere}{=} 2 + \frac{0.589 Ra_D^{1/4}}{\left[1+\left(0.469/Pr\right)^{9/16}\right]^{4/9}}$

Boiling:

Basic Equation: $q_{boiling} = h_{boiling} A \Delta T_e = h_{boiling} A \left(T_s - T_{sat}\right)$

Nucleate Boiling: $q_s'' \underset{boiling}{\overset{nucleate}{=}} \mu_l h_{fg} \left[\frac{g\left(\rho_l - \rho_v\right)}{\sigma}\right]^{1/2} \left[\frac{c_{p,l} \Delta T_e}{C_{sf} h_{fg} Pr_l^n}\right]^3$;

$q_{max}'' \underset{boiling}{\overset{nucleate}{=}} \frac{\pi}{24} h_{fg} \rho_v \left[\frac{\sigma g\left(\rho_l - \rho_v\right)}{\rho_v^2}\right]^{1/4}$

Minimum Heat Flux: $q_{min}'' = 0.09 h_{fg} \rho_v \left[\frac{\sigma g\left(\rho_l - \rho_v\right)}{\left(\rho_l + \rho_v\right)^2}\right]^{1/4}$

Film Boiling: $\overline{Nu_D} = \frac{\overline{h_D} D}{k_v} \underset{boiling}{\overset{film}{=}} C \left[\frac{g\left(\rho_l - \rho_v\right) h_{fg}' D^3}{\nu_v k_v \left(T_s - T_{sat}\right)}\right]^{1/4}$;

$h_{fg}' = h_{fg} + 0.8 C_{p,v} \left(T_s - T_{sat}\right)$; $C \overset{cylinder}{=} 0.62$; $C \overset{sphere}{=} 0.67$

Condensation:

Basic Equation: $q_{condensation} = h_{condensation} A \Delta T_d = h_{condensation} A \left(T_{sat} - T_s\right)$; $\dot{m}_{condensation} = \frac{q_{condensation}}{h_{fg}'}$

Film Condensation:

Vertical Flat Plate: $\overline{Nu_L} = \frac{\overline{h_L} L}{k_l} \underset{film\ condensation}{\overset{laminar}{=}} 0.943 \left[\frac{g\left(\rho_l - \rho_v\right) h_{fg}' L^3}{\nu_l k_l \left(T_{sat} - T_s\right)}\right]^{1/4}$; $h_{fg}' = h_{fg} + 0.68 C_{p,l} \left(T_{sat} - T_s\right)$

Heat Exchangers:

$$\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}} \quad q = UA \Delta T_{LMTD}$$

Heat exchanger effectiveness: $\varepsilon = \frac{q}{q_{\max}} = \frac{q}{C_{\min}\left(T_{hi} - T_{ci}\right)}$

Number of transfer units: NTU = $\dfrac{UA}{C_{\min}}$

Radiation

Two Surface Interaction: $d\omega_{2-1} = \dfrac{dA_{2,normal}}{r^2} = \dfrac{A_2 cos\theta_2}{r^2}$; $q_{1-2} = I_e\left(A_1 cos\theta_1\right) d\omega_{2-1}$

Emissive Power: $E_\lambda(\lambda) = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi) cos\theta sin\theta d\theta$; $E = \int_0^\infty E_\lambda(\lambda) d\lambda$

$E_\lambda(\lambda) \overset{diffuse\ emitter}{=} \pi I_{\lambda,e}(\lambda)$; $E \overset{diffuse\ emitter}{=} \pi I_e$

Irradiation: $G_\lambda(\lambda) = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) cos\theta sin\theta d\theta$; $G = \int_0^\infty G_\lambda(\lambda) d\lambda$

$G_\lambda(\lambda) \overset{diffuse\ irradiation}{=} \pi I_{\lambda,i}(\lambda)$; $G \overset{diffuse\ irradiation}{=} \pi I_i$

Radiosity: $J_\lambda(\lambda) = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\phi) cos\theta sin\theta d\theta$; $J = \int_0^\infty J_\lambda(\lambda) d\lambda$

$J_\lambda(\lambda) \underset{diffuse\ reflector}{\overset{diffuse\ emitter}{=}} \pi I_{\lambda,e+r}(\lambda)$; $J \underset{diffuse\ reflector}{\overset{diffuse\ emitter}{=}} \pi I_{e+r}$

Black Body Emission: $E_{\lambda,b}(\lambda,T) \overset{Black\ Body}{=} \dfrac{C_1}{\lambda^5\left[\exp\left(\dfrac{C_2}{\lambda T}\right)-1\right]}$; $E_b(T) \overset{Black\ Body}{=} \int_0^\infty E_{\lambda,b}(\lambda,T) d\lambda \overset{Black\ Body}{=} \sigma T^4$;

$E_b \overset{Black\ Body}{=} \pi I_b$;

$\lambda_{max} T \overset{Black\ Body}{=} 2898 \mu mK$ Wein's displacement law

Radiative Properties:

Emissivity: $\varepsilon_{\lambda,\theta} = \dfrac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{\lambda,eb}(\lambda,T)}$;

$$\varepsilon_\lambda = \frac{E_\lambda}{E_{\lambda,b}} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi,T) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,eb}(\lambda,T) cos\theta sin\theta d\theta}; \quad \varepsilon = \frac{\int_0^\infty E_\lambda(\lambda,T) d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda,T) d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda,T) d\lambda}{\sigma T^4}$$

Absorptivity: $\alpha_{\lambda,\theta} = \dfrac{I_{\lambda,i\ absorbed}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$; $\alpha_\lambda = \dfrac{G_{\lambda,absorbed}(\lambda)}{G_\lambda(\lambda)} = \dfrac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i\ absorbed}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}$;

$\alpha = G_{absorbed} / G$; $\alpha = \dfrac{\int_0^\infty \alpha_\lambda G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$

Reflectivity: $\rho_{\lambda,\theta} = \dfrac{I_{\lambda,i\ reflected}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$; $\rho_\lambda = \dfrac{G_{\lambda,reflected}(\lambda)}{G_\lambda(\lambda)} = \dfrac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i\ reflected}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}$;

$\rho = G_{reflected}/G$; $\rho = \dfrac{\int_0^\infty \rho_\lambda G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$

Transmissivity: $\tau_{\lambda,\theta} = \dfrac{I_{\lambda,i\ transmitted}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$;

$\tau_\lambda = \dfrac{G_{\lambda,transmitted}(\lambda)}{G_\lambda(\lambda)} = \dfrac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i\ transmitted}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}$; $\tau = G_{transmitted}/G$; $\tau = \dfrac{\int_0^\infty \tau_\lambda G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$

Semi-transparent Surface: $\alpha_\lambda + \rho_\lambda + \tau_\lambda \overset{semi-transparent}{=} 1$; $\alpha + \rho + \tau \overset{semi-transparent}{=} 1$

Opaque Surface: $\alpha_\lambda + \rho_\lambda \overset{opaque}{=} 1$; $\alpha + \rho \overset{opaque}{=} 1$

Kirchhoff's Law: $\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}$

Gray Surface: $\varepsilon_\lambda \neq f(\lambda)$; $\alpha_\lambda \neq f(\lambda)$;

Diffuse-Gray Surface: $\varepsilon = \alpha$

Useful Constants

σ = Stefan-Boltzmann's Constant = $5.67 \times 10^{-8} \dfrac{\text{W}}{\text{m}^2\text{K}^4}$

Geometry

Cylinder: $A = 2\pi rl$; $V = \pi r^2 l$

Sphere: $A = 4\pi r^2$; $V = \dfrac{4}{3}\pi r^3$

Triangle: A= bh/2 b:base h: height