ME 315
Exam 3
8:00 -9:00 PM
Thursday, April 16, 2009

- This is a closed-book, closed-notes examination. *There is a formula sheet at the back.*
- You must turn off all communications devices before starting this exam, and leave them off for the entire exam.
- Please write legibly and show all work for your own benefit. Please show your final answers in the boxes provided.
- State all assumptions.
- Please arrange all your sheets in the correct order. Make sure they are all included.

Name: ___________________________________
Last First

CIRCLE YOUR DIVISION

Div. 1 (9:30 am)
Prof. Murthy

Div. 2 (12:30 pm)
Prof. Choi

Problem	Score
1 (30 Points)	
2 (30 Points)	
3 (40 Points)	
Total (100 Points)	

1. (30 points) Consider a heat exchanger with a heat exchange area of 30 m^2 and operating under the conditions listed in the table below.

	Hot Fluid	Cold Fluid
Heat Capacity C (kW/K)	5	2
Inlet temperature ^oC	60	45
Outlet temperature ^oC	--	57.5

(i) Determine the outlet temperature of the hot stream in ^oC.

T_{ho} =

(ii) Is the heat exchanger operating in

a. Parallel flow
b. Counterflow
c. Can't tell

Circle only one answer and explain your choice.

Explanation:

(iii) Calculate the overall heat transfer coefficient U (W/m^2K).

U =

(iv) What would the effectiveness be if the heat exchanger were infinitely long?

ε =

Solution

(i)

$$q = \dot{m}\, c_p\, \Delta T = \dot{m}_h\; c_{p,h}\,(T_{h,i} - T_{h,o}) = \dot{m}_c\; c_{p,c}\,(T_{c,o} - T_{c,i})$$
$$= C_h\,(T_{h,i} - T_{h,o}) = C_c\,(T_{c,o} - T_{c,i})$$

$$\therefore q = (\,2\,kW\,/\,K)(57.5°C - 45°C) = (\,5\,kW\,/\,K)(60°C - T_{h,o}) = 25\,kW$$

$$\boxed{\therefore T_{h,o} = 55\;°C}$$

(ii)

$$T_{c,o} = 57.5°C \quad T_{h,o} = 55°C \quad \Rightarrow \quad T_{c,o} > T_{h,o}$$

$$\boxed{\therefore Counterflow}$$

(iii)

$$q = U\,A\,\Delta T_{lm} = C_h\,(T_{h,i} - T_{h,o}) = C_c\,(T_{c,o} - T_{c,i}) = 25\,kW$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{ln\,(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1 = T_{h,i} - T_{c,o} = 60°C - 57.5°C = 2.5°C$$
$$\Delta T_2 = T_{h,o} - T_{c,i} = 55°C - 45°C = 10°C$$

$$\therefore \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{ln\,(\Delta T_1/\Delta T_2)} = \frac{2.5°C - 10°C}{ln\,(2.5°C/10°C)} = 5.41011°C$$

$$\therefore q = U\,A\,\Delta T_{lm} = U\,(30\,m^2)(5.41011°C) = 25\,kW$$

$$\boxed{\therefore U = 0.154033\;\frac{kW}{m^2 - K} = 154.033\;\frac{W}{m^2 - K}}$$

(iv)

$$\varepsilon = \frac{q}{q_{\max}}$$

$$q_{\max} = C_{\min}\,(T_{h,i} - T_{h,i}) = (2\,kW/K)(60°C - 45°C) = 30\,kW$$

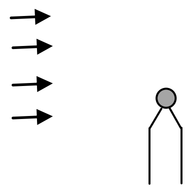
$$As\ L \rightarrow \infty,\ \ T_{c,o} = T_{h,i} = 60°C$$

$$q = C_c\,(T_{c,o} - T_{c,i}) = (2\,kW/K)(60°C - 45°C) = 30\,kW$$

$$\boxed{\therefore \varepsilon = \frac{q}{q_{\max}} = \frac{30\,kW}{30\,kW} = 1}$$

2. (30 points) You are measuring the combustion gas temperature at the turbine inlet in a gas turbine engine using a thermocouple junction. The thermocouple has a diameter of 1 mm, and is initially at T_i = 25 °C. You may neglect radiation in the following analysis.

Hot combustion gas
(evaluated at the
free stream temperature)
k = 0.04 W/mK
ρ = 0.6 kg/m^3
c_p = 1000 J/kgK
ν = 50 x 10^{-6} m^2/s
U = 20 m/s



Thermocouple
k = 300 W/mK
ρ = 9000 kg/m^3
c_p = 400 J/kgK

μ_s = 25 x 10^{-6} kg/ms (gas viscosity evaluated at the thermocouple surface temperature, T_s)

(i) Determine the average convective heat transfer coefficient, $\bar{h}$.

$\bar{h}$ = W/m^{2}K

(ii) The temperature measured at the thermocouple centerline after 3 sec is 900 °C. Determine the combustion gas temperature, T_∞.

T_∞ = K

Solution

(i)

$$\overline{Nu_D} = 2 + (0.4\,Re_D{}^{1/2} + 0.06\,Re_D{}^{2/3})\,Pr^{0.4}\,(\frac{\mu}{\mu_s})^{1/4}$$

$$Re_D = \frac{\rho U L_c}{\mu} = \frac{\rho U D}{\mu} = \frac{\rho U}{\nu} = \frac{(20\,m/s)\,(0.001m)}{50\times10^{-6}\;m^2/s} = 400$$

$$Pr = \frac{\nu}{\alpha} = \nu\frac{\rho c_p}{k} = (50\times10^{-6}\;m^2/s)\frac{(0.6\,kg/m^3)\,(1000\,J/kg-K)}{0.04\;W/m-K} = 0.75$$

$$\mu = \nu\cdot\rho = (50\times10^{-6}\;m^2/s)\cdot(0.6\,kg/m^3) = 3\times10^{-5}\;kg/m-s$$

$$\therefore \overline{Nu_D} = 2 + (0.4\,Re_D{}^{1/2} + 0.06\,Re_D{}^{2/3})\,Pr^{0.4}\,(\frac{\mu}{\mu_s})^{1/4}$$

$$= 2 + (0.4\cdot 400^{1/2} + 0.06\cdot 400^{2/3})(0.75^{0.4})(\frac{3\times10^{-5}\;kg/m-s}{25\times10^{-6}\;kg/m-s})^{1/4} = 12.502$$

$$\overline{Nu_D} = \frac{\overline{h}\cdot D}{k}$$

$$\boxed{\therefore \overline{h} = \overline{Nu_D}\cdot\frac{k}{D} = 12.502\cdot\frac{0.04\,W/m-K}{0.001m} = 500.063\,W/m^2-K}$$

(ii)

$$Bi = \frac{\bar{h} L_c}{k_{solid}} \qquad L_c = \frac{r_o}{3} = \frac{0.001 m}{6} \sqrt{b^2 - 4ac}$$

$$Bi = \frac{\bar{h} L_c}{k_{solid}} = \frac{(500.063\, W/m^2 - K)\cdot(\frac{0.001 m}{6})}{300\, W/m - K} = 0.000278 < 0.1$$

$\therefore$ Lumped capacitance analysis is valid.

$$\frac{T - T_\infty}{T_i - T_\infty} = exp(-\frac{t}{\tau_t})$$

$$\tau_t = \frac{\rho \cdot V \cdot c_p}{\bar{h} \cdot A_s} \qquad V = \frac{4}{3}\cdot \pi \cdot r^3 \qquad A_s = 4 \cdot \pi \cdot r^2$$

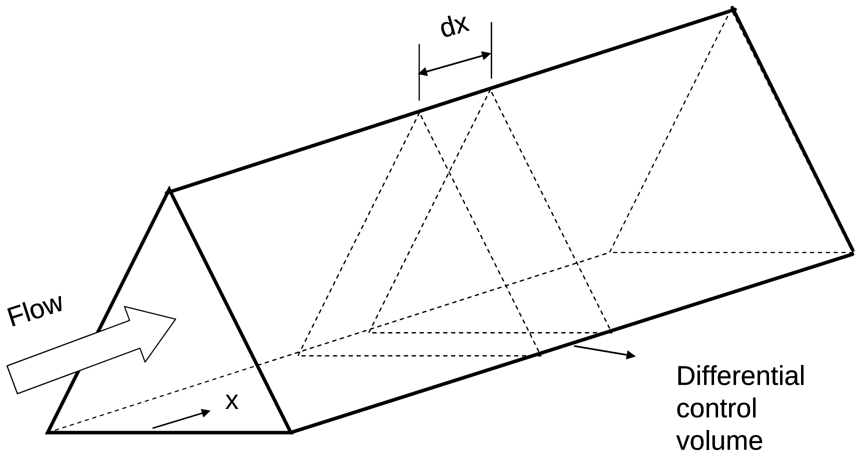
$$\therefore \tau_t = \frac{\rho \cdot V \cdot c_p}{\bar{h} \cdot A_s} = \frac{\rho \cdot (\frac{4}{3}\cdot \pi \cdot r^3)\cdot c_p}{\bar{h}\cdot(4 \cdot \pi \cdot r^2)} = \frac{\rho \cdot r \cdot c_p}{3 \cdot \bar{h}} = \frac{(9000\, kg/m^3)\cdot(\frac{0.001}{2} m)\cdot(400\, J/kg - K)}{3 \cdot (500.063\, W/m^2 - K)} = 1.20\, sec$$

$At\ t = 3\ sec, T = 900\ °C$

$$\therefore \frac{T - T_\infty}{T_i - T_\infty} = \frac{900\ °C - T_\infty}{25\ °C - T_\infty} = exp(-\frac{3\ sec}{1.20\ sec})$$

$$\boxed{\therefore T_\infty = 978.247\ °C}$$

3. (40 points) Fluid enters a duct of triangular cross-section with a mass flow rate of $\dot{m}$. The cross-section is an equilateral triangle of side *s*. The fluid is heated at the walls with a heat flux $q'' = ax + b$ (W/m^2) into the domain, where *x* is measured from the entrance to the duct. Furthermore, a chemical reaction in the volume of the fluid causes a constant volumetric heat generation rate of $\dot{q}$ W/m^3. The inlet bulk temperature of the fluid is T_{mi}. You are given that the fluid has a constant density ρ and a constant specific heat C_p.



(a) By considering the differential control volume shown in the figure, write an energy balance to derive a <u>symbolic expression</u> for $\frac{dT_m}{dx}$ as a function of *a, b, s, x,* $\dot{m}$, $\dot{q}$ and the physical properties of the fluid.

$\frac{dT_m}{dx} =$

(b) Derive a <u>symbolic expression</u> for the variation of T_m with x in terms of *a, b, s, x,* $\dot{m}$, $\dot{q}$, the inlet bulk temperature T_{mi} and the physical properties of the fluid.

$T_m(x) =$

(c) Is the expression you derived in part (b) for $T_m(x)$

(i) always valid for this problem?

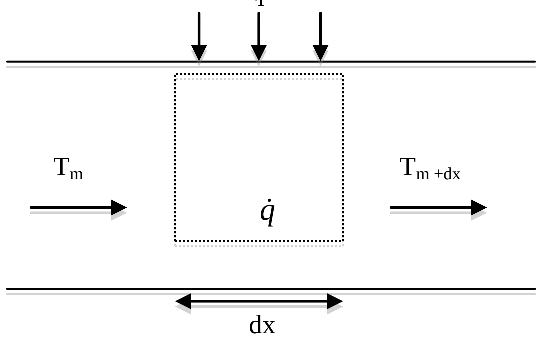
(ii) only valid for this problem if the flow is fully developed but the temperature is developing?

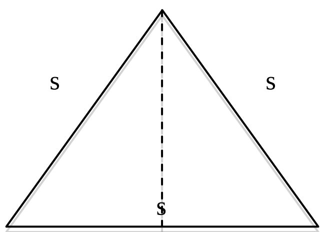
(iii) only valid for this problem if the flow and temperature are fully-developed?

Circle only one answer, and justify your choice.

Solution

(a)





$\frac{\sqrt{3}s}{2}$

Perimeter = 3s Area = $\frac{\sqrt{3}\,s^2}{4}$

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{store}$$

$$\dot{E}_{in} = \dot{m}\, c_p\, T_m + q''\cdot(3s)\cdot dx = \dot{m}\, c_p\, T_m + (a\,x + b)\cdot(3s)\cdot dx$$

$$\dot{E}_{out} = \dot{m}\, c_p\, T_{m+dx} = \dot{m}\, c_p\, (T_m + \frac{d\,T_m}{dx}\,dx)$$

$$\dot{E}_{gen} = \dot{q}\cdot V = \dot{q}\,A\cdot dx = \dot{q}\,\frac{\sqrt{3}\,s^2}{4}\cdot dx$$

$$\dot{E}_{store} = 0$$

$$\therefore\ \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{m}\, c_p\, T_m + (a\,x + b)\cdot(3s)\cdot dx - \dot{m}\, c_p\, (T_m + \frac{d\,T_m}{dx}\,dx) + \dot{q}\,\frac{\sqrt{3}\,s^2}{4}\cdot dx = 0$$

$$(3s)\cdot(a\,x + b)\cdot dx - \dot{m}\, c_p\,\frac{d\,T_m}{dx}\,dx + \frac{\sqrt{3}\,s^2\,\dot{q}}{4}\cdot dx = 0$$

$$\dot{m}\, c_p\,\frac{d\,T_m}{dx} = (3s)\cdot(a\,x + b) + \frac{\sqrt{3}\,s^2\,\dot{q}}{4}$$

$$\boxed{\therefore \frac{d\,T_m}{dx} = \frac{(3s)\cdot(a\,x + b)}{\dot{m}\, c_p} + \frac{\sqrt{3}\,s^2\,\dot{q}}{4\dot{m}\, c_p}}$$

(b)

$$\frac{dT_m}{dx} = \frac{(3s)\cdot(a\,x+b)}{\dot{m}\,c_p} + \frac{\sqrt{3}\,s^2\,\dot{q}}{4\dot{m}\,c_p}$$

$$\int_{T_{m,i}}^{T_m} d\,T_m = \int_0^x \left[\frac{(3s)\cdot(a\,x+b)}{\dot{m}\,c_p} + \frac{\sqrt{3}\,s^2\,\dot{q}}{4\dot{m}\,c_p}\right]dx$$

$$T_m - T_{m,i} = \left[\frac{(3s)}{\dot{m}\,c_p}\cdot\left(\frac{a\,x^2}{2} + b\,x\right) + \frac{\sqrt{3}\,s^2\,\dot{q}}{4\dot{m}\,c_p}\,x\right]\Bigg|_0^x = \frac{3s}{\dot{m}\,c_p}\cdot\left(\frac{a\,x^2}{2} + b\,x\right) + \frac{\sqrt{3}\,s^2\,\dot{q}}{4\dot{m}\,c_p}\,x$$

$$\boxed{\therefore T_m = T_{m,i} + \frac{3s}{\dot{m}\,c_p}\cdot\left(\frac{a\,x^2}{2} + b\,x\right) + \frac{\sqrt{3}\,s^2\,\dot{q}}{4\dot{m}\,c_p}\,x}$$

(c) It is **ALWAYS VALID** for the problem. The expression was derived using a simple energy balance. Therefore, no assumptions about fully developed flow or heat transfer are necessary. In fact, if q” varies with x, no thermally fully developed flow is possible.