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CIRCLE YOUR DIVISION:

Div. 1 (9:30 am)	**Div. 2 (11:30 am)**	**Div. 3 (2:30 pm)**
Prof. Ruan	**Prof. Naik**	**Mr. Singh**

School of Mechanical Engineering
Purdue University
ME315 Heat and Mass Transfer

Exam #3

Wednesday, November 17, 2010

Instructions:

- Write your name on each page
- Closed-book exam – a list of equations is given
- Please write legibly and show all work for your own benefit. Write on one side of the page only
- Keep all pages in order
- <u>You are asked to write your answers to sub-problems in designated areas. Only the work in its designated area will be graded.</u>

Performance

1	**30**	
2	**35**	
3	**35**	
Total	**100**	

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Problem 1 [30 pts]

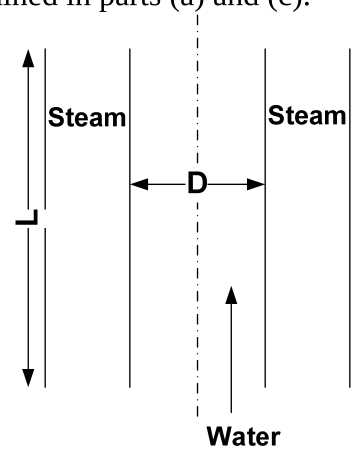
Consider a well-insulated, concentric tube, counter-flow heat exchanger used to heat cold water. Saturated steam at a temperature of T_{sat} condenses in the outer tube at mass rate of $\dot{m}_o$. Cold water enters the inner tube of diameter D and length L at a temperature of T_i, with a mass flow rate $\dot{m}_i$, and an average heat transfer coefficient h_i. The latent heat of vaporization of water is h_{fg} and the average specific heat of water is c_p. Assume that the inner tube has negligible thickness. Neglect any surface fouling effects.

(a) Find an expression for the outlet temperature T_o of water in the inner tube.

(b) Sketch the temperature variation of the two fluids along the length of the heat exchanger and clearly label the temperatures.

(c) Find an expression for the effectiveness ε of the heat exchanger only in terms of T_{sat}, T_i, and T_o. Write an expression for NTU in terms of ε.

(d) Express the average heat transfer coefficient for steam condensation h_o only in terms of known quantities and/or those obtained in parts (a) and (c).

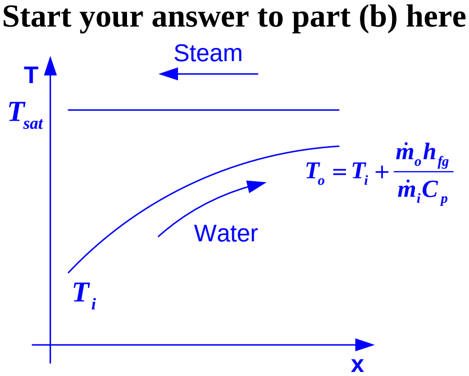


Start your answer to part (a) here [6 pts]:

Assuming heat lost by condensing steam to be equal to the heat gained by cold water, we can write: $\dot{m}_o h_{fg} = \dot{m}_i C_p \left(T_o - T_i\right)$

Outlet temperature of water: $\underline{\boldsymbol{T_o = T_i + \dfrac{\dot{m}_o h_{fg}}{\dot{m}_i C_p}}}$

Start your answer to part (b) here [6 pts]:



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Problem 1 – cont.

Start your answer to part (c) here [12 pts]:

For condensing steam in the outer tube, temperature does not change $\Rightarrow \; C_h = \dot{m}_o C_{p,steam} \to \infty$

Therefore, in this heat exchanger: $C_h = C_{max} \to \infty$ and $C_c = C_{min} = \dot{m}_i C_p \Rightarrow C_r = \dfrac{C_{min}}{C_{max}} = 0$

Effectiveness: $\varepsilon = \dfrac{q}{q_{max}} = \dfrac{q}{C_{min}\left(T_{hi} - T_{ci}\right)} = \dfrac{\dot{m}_i C_p \left(T_o - T_i\right)}{\dot{m}_i C_p \left(T_{sat} - T_i\right)} \Rightarrow \underline{\boldsymbol{\varepsilon = \dfrac{T_o - T_i}{T_{sat} - T_i}}}$; T_o known from (a)

For $C_r = 0 : \varepsilon = 1 - \exp\left(-NTU\right) \Rightarrow \underline{\boldsymbol{NTU = -\ln\left(1-\varepsilon\right)}}$

Start your answer to part (d) here [6 pts]:

Overall heat transfer coefficient: $\dfrac{1}{UA} = \dfrac{1}{h_i A_i} + \cancel{R}_{f,i} + \dfrac{\ln\left(\dfrac{\cancel{r_o}}{r_i}\right)}{2\pi k L} + \cancel{R}_{f,i} + \dfrac{1}{h_o A_o}$ $;\, r_i = r_o ;\, A = A_i = A_o$

$\Rightarrow \dfrac{1}{U} = \dfrac{1}{h_i} + \dfrac{1}{h_o} ;\, NTU = \dfrac{UA}{C_{min}} \Rightarrow -\ln\left(1-\varepsilon\right) = \dfrac{U\left(\pi D L\right)}{\dot{m}_i C_p} \Rightarrow U = \dfrac{-\ln\left(1-\varepsilon\right)\dot{m}_i C_p}{\pi D L}$

Heat transfer coefficient for steam condensation: $\underline{\boldsymbol{h_o = \dfrac{1}{\dfrac{1}{U} - \dfrac{1}{h_i}}}}$; U and h_i are known

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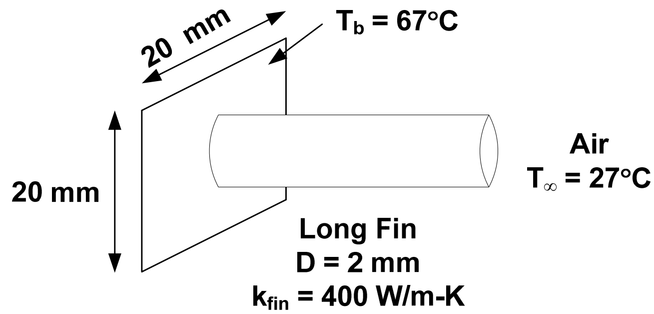
Problem 2 [35 pts]

Consider a long pin fin attached to a vertical square chip (20 mm × 20 mm). The chip has a uniform surface temperature of 67°C while the surrounding stagnant air is at 27°C.



(a) Calculate the average convective heat transfer coefficient (W/m^2-K) on the fin surface. Since the surface temperature varies, you can use a representative fin surface temperature equal to the average of the base and tip temperatures in this calculation.

(b) Calculate the average convective heat transfer coefficient (W/m^2-K) on the unfinned base. Comment on whether it is close to the value obtained in part (a), which is an assumption often made in fin analysis.

(c) Determine the total heat dissipation rate (W) from the fin and the unfinned chip.

At T = 300 K

ν_{air} = 15.89 × 10^{-6} m^2/s α_{air} = 22.5 × 10^{-6} m^2/s k_{air} = 0.0263 W/m-K

At T = 310 K

ν_{air} = 16.90 × 10^{-6} m^2/s α_{air} = 24.0 × 10^{-6} m^2/s k_{air} = 0.0270 W/m-K

At T = 320 K

ν_{air} = 17.90 × 10^{-6} m^2/s α_{air} = 25.5 × 10^{-6} m^2/s k_{air} = 0.0278 W/m-K

Start your answer to part (a) here [15 pts]:

Average fin surface temperature: $T_{s,fin} = \dfrac{T_b + T_L}{2} = \dfrac{T_b + T_\infty}{2} = 47^\circ\text{C}$

For the fin surface, film temperature: $T_{film} = \dfrac{T_{s,fin} + T_\infty}{2} = 37^\circ\text{C} = 310\ \text{K}$

ν_{air} = 16.90 × 10^{-6} m^2/s, α_{air} = 24.0 × 10^{-6} m^2/s, k_{air} = 0.0270 W/m-K, $\beta = \dfrac{1}{T_{film}} = \dfrac{1}{310}\text{K}^{-1}$

For the cylindrical fin surface, the Rayleigh number: $Ra_D = \dfrac{g\beta\left(T_{s,fin} - T_\infty\right)D^3}{\nu\alpha} = 12.48$

Using Table 9.1, $\overline{Nu_D} = \dfrac{\overline{h_{fin}}D}{k} = 1.02 Ra_D^{0.148} = 1.48$

Average convective heat transfer coefficient on the fin surface: $\boxed{\overline{h_{fin}} = \mathbf{20}\dfrac{\mathbf{W}}{\mathbf{m^2\text{-}K}}}$

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Problem 2 - cont.

Start your answer to part (b) here [10 pts]:

For the unfinned base, film temperature: $T_{film} = \dfrac{T_b + T_\infty}{2} = 47^\circ\text{C} = 320\ \text{K}$

$\nu_{air} = 17.90 \times 10^{-6}\ \text{m}^2/\text{s}$, $\alpha_{\text{air}} = 25.5 \times 10^{-6}\ \text{m}^2/\text{s}$, $k_{air} = 0.0278$ W/m-K, $\beta = \dfrac{1}{T_{film}} = \dfrac{1}{320}\text{K}^{-1}$

For the unfinned base, the Rayleigh number: $Ra_L = \dfrac{g\beta\left(T_b - T_\infty\right)L^3}{\nu\alpha} = 21,492$

$$\overline{Nu_L} = \frac{\overline{h_{base}}L}{k} = 0.59Ra_L^{1/4} = 7.15$$

Average convective heat transfer coefficient on the unfinned base: $\boldsymbol{\overline{h_{base}} = 9.95\dfrac{\textbf{W}}{\textbf{m}^2\textbf{-K}}}$

The two convective heat transfer coefficients are different since the development of the free convection boundary layer is not the same on cylindrical and vertical surface.

Start your answer to part (c) here [10 pts]:

Heat transfer rate through the fin: $q_{fin} = \sqrt{\overline{h_{fin}}Pk_{fin}A_c}\theta_b$

$P = \pi D = 6.28\times10^{-3}\text{m}$, $A_c = \dfrac{\pi}{4}D^2 = 3.14\times10^{-6}\text{m}^2$, $\theta_b = T_b - T_\infty = 40\ \text{K}$

$q_{fin} = 0.5024\ \text{W}$

Heat transfer rate through the unfinned base: $q_{base} = \overline{h_{base}}\left(A_{base} - A_c\right)\theta_b$; $A_{base} = 400\times10^{-6}\text{m}^2$

$q_{base} = 0.1580\ \text{W}$

Total heat transfer rate through the chip: $q_{chip} = q_{base} + q_{fin}$ i.e. $\boldsymbol{q_{chip} = 0.66\ \textbf{W}}$

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Problem 3 [35 pts]

A fluid is introduced at flow rate $\dot{m}$ and inlet mean temperature $T_{m,i}$ through a circular tube of diameter D and length L. The tube is divided into two equal sections. The fluid is subjected to a constant heat flux $q_s" = a$ for the first half while the tube wall of the second half is maintained at a constant surface temperature. It is found that the outlet mean temperature $T_{m,o}$ is the same as the inlet mean temperature $T_{m,i}$. Assume that the flow is turbulent and fully developed everywhere inside the tube. c_p, k, and μ of the fluid are known. You may treat the quantities obtained in earlier parts as known parameters in the later parts.

(a) Recommend correlations for the convective heat transfer coefficient h inside the tube for the two halves of the tube.

(b) Derive an expression for the mean fluid temperature $T_m(x)$ along the first half of the tube.

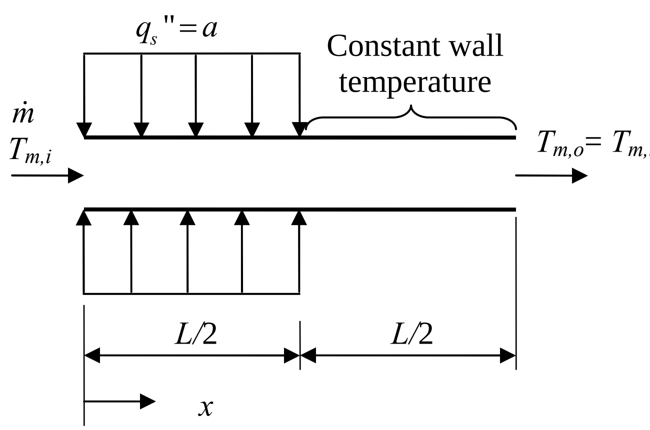
(c) Derive an expression for the tube surface temperature $T_s(x)$ along the first half of the tube.

(d) Obtain an expression for constant wall temperature $T_{s,c}$ for the second half of the tube.

(e) Obtain an expression for the mean fluid temperature $T_m(x)$ for the second half of the tube.

(f) Qualitatively sketch the variations of $T_m(x)$ and $T_s(x)$ for $0 < x < L$.

Start your answer to part (a) here [8 pts]:



For turbulent, fully-developed flow (either constant flux or constant temperature):

$$Nu_D = \frac{hD}{k} = 0.023 Re_D^{4/5} Pr^n$$

For the first half, fluid is being heated: $\frac{h_1 D}{k} = 0.023 Re_D^{4/5} Pr^{0.4}$

For the second half, fluid is being cooled: $\frac{h_2 D}{k} = 0.023 Re_D^{4/5} Pr^{0.3}$

where, $Re_D = \frac{u_m D\rho}{\mu} = \left(\frac{\dot{m}}{\frac{\pi}{4}D^2\rho}\right)\frac{D\rho}{\mu} = \frac{4\dot{m}}{\pi D\mu}$; $Pr = \frac{\nu}{\alpha} = \frac{\mu}{\rho}\frac{\rho C_p}{k} = \frac{\mu C_p}{k}$

Start your answer to part (b) here [6 pts]:

Considering energy balance for the first half: $\dot{m}C_p dT_m = q_s'' P dx \Rightarrow dT_m = \frac{aPdx}{\dot{m}C_p}$

Integrating: $\int_{T_{m,i}}^{T_m(x)} dT_m = \frac{aP}{\dot{m}C_p}\int_0^x dx \Rightarrow \underline{\boldsymbol{T_m(x) = T_{m,i} + \frac{aP}{\dot{m}C_p}x}}$; $P = \pi D$

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Problem 3 - cont.

Start your answer to part (c) here [4 pts]:

Considering heat flux at any section: $q_s'' = h_1\left[T_s(x) - T_m(x)\right] \Rightarrow \boldsymbol{T_s(x) = T_m(x) + \frac{a}{h_1}}$; $T_m(x)$ and h_1 from (b) and (a), respectively

Start your answer to part (d) here [6 pts]:

For the second half: $$\frac{T_{s,c} - T_{m,o}}{T_{s,c} - T_m\left(\frac{L}{2}\right)} = \exp\left(-\frac{Ph_2}{\dot{m}C_p}\frac{L}{2}\right) \Rightarrow \boldsymbol{T_{s,c} = \frac{T_{m,i} - T_m\left(\frac{L}{2}\right)\exp\left(-\frac{Ph_2}{\dot{m}C_p}\frac{L}{2}\right)}{1 - \exp\left(-\frac{Ph_2}{\dot{m}C_p}\frac{L}{2}\right)}}$$

where $T_m\left(\frac{L}{2}\right) = T_{m,i} + \frac{aP}{\dot{m}C_p}\frac{L}{2}$, $P = \pi D$, and h_2 from (a)

Start your answer to part (e) here [5 pts]:

For the second half: $$\frac{T_{s,c} - T_m(x)}{T_{s,c} - T_m\left(\frac{L}{2}\right)} = \exp\left(-\frac{Ph_2}{\dot{m}C_p}\left(x - \frac{L}{2}\right)\right) \Rightarrow$$

$\boldsymbol{T_m(x) = T_{s,c} - \left(T_{s,c} - T_m\left(\frac{L}{2}\right)\right)\exp\left(-\frac{Ph_2}{\dot{m}C_p}\left(x - \frac{L}{2}\right)\right)}$; where $P = \pi D$, $T_{s,c}$ and h_2 from (d) and (a), respectively

Start your answer to part (f) here [6 pts]: