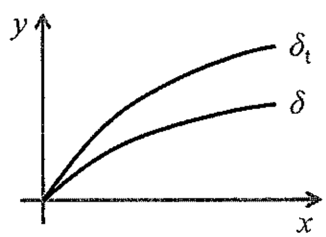
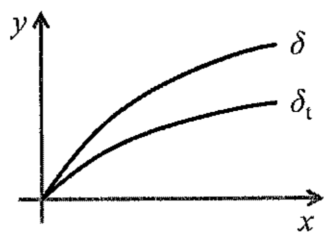
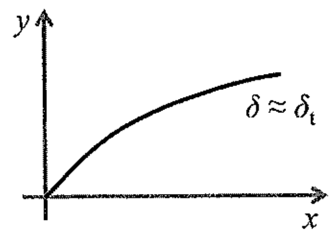
(1) R134a is a widely used refrigerant with a Prandtl number of $Pr = 3.4$ at 300 K. Consider an experiment of laminar, external flow of R134a over an isothermal flat plate ($T_s = 300$ K). Which of the following graph correctly depicts the relative thicknesses of the velocity boundary layer (δ) and thermal boundary layer (δ_t)?



(1)



(2)



(3)

Solution:

For laminar, external flow over an isothermal flat plate, $\dfrac{\delta}{\delta_t} \approx \mathrm{Pr}^{1/3}$. In this problem, since $\mathrm{Pr} > 1$, $\delta > \delta_t$. The answer is (2).

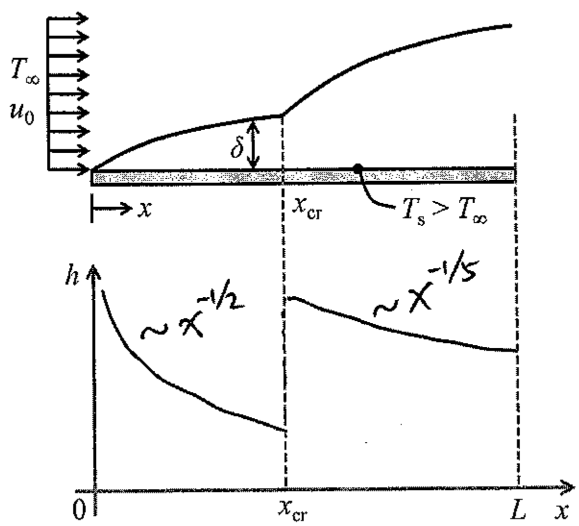
Grading:

If the answer is correct, then give full points.

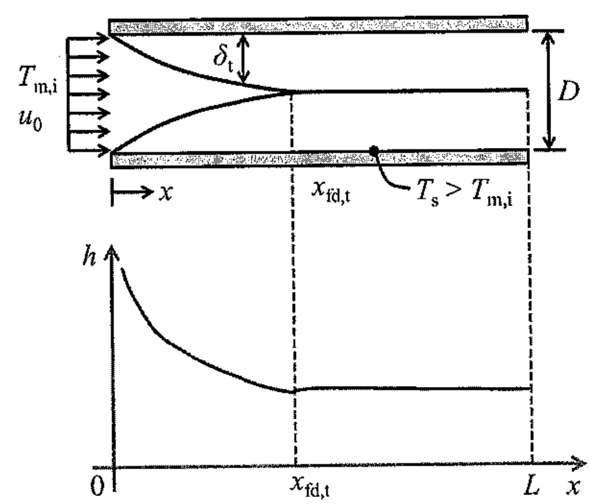
If the answer is wrong, then give partial credits based on the student's reasoning process.

(2) Consider (1) external flow over an isothermal flat plate and (2) internal flow in an isothermal pipe. Based on the boundary layers given below, sketch the corresponding heat transfer coefficient profiles and indicate important characteristics.

(1) External flow over an isothermal flat plate



(2) Internal flow in an isothermal pipe



Solution:

Heat transfer coefficient is inversely proportional to the boundary layer thickness ($h \sim 1/\delta_t$).

(1) For external flow over an isothermal flat plate.

At $x < x_{cr}$, laminar B.L., $h \sim x^{-1/2}$.

At $x = x_{cr}$, transition, laminar B.L. → turbulent B.L., a jump up of h.

At $x > x_{cr}$, turbulent B.L., $h \sim x^{-1/5}$.

(2) For internal flow in an isothermal pipe.

At $x < x_{fd,t}$, thermally developing flow, similar to external flow, $h \sim x^{-1/2}$ (approximately).

At $x \geq x_{fd,t}$, thermally fully developed flow, h becomes a constant ($Nu = 3.66$).

Grading:

If the sketches are all correct, then give full points.

If the sketches are partially correct, then given partial credits.

If the sketches are totally wrong, then give partial credits based on the student's reasoning process.

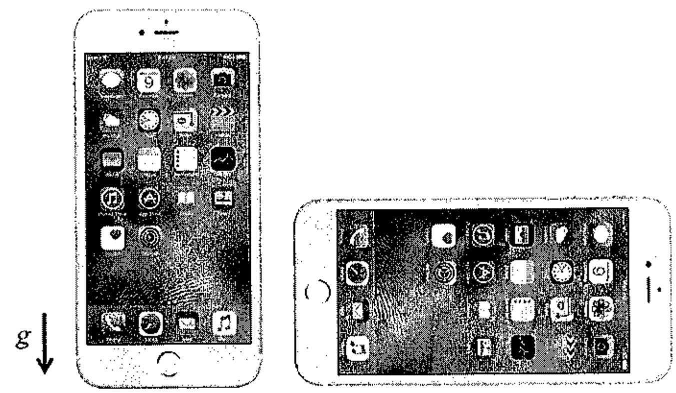
(3) Consider a free convection cooling of a vertically placed iPhone 6 in two configurations (A) and (B). Assume that the iPhone 6 has a constant surface temperature T_s and that the boundary layers are laminar. It is known that the correlation of average Nusselt number for laminar free convection on vertical plate has the form $\overline{Nu} = \frac{\bar{h} L}{k} = C Ra_L^{1/4}$, where C is a constant and L is the height of the plate. You may also assume that the edge effects are negligible. What is the relation between the cooling rates of the two configurations?

(1) $q_A > q_B$

(2) $q_A < q_B$

(3) $q_A = q_B$

(4) None of the above.



(A) (B)

Solution:

Newton's cooling law: $q = hA(T_s - T_\infty)$.

A and $(T_s - T_\infty)$ are the same for both configurations (A) and (B). So $q \sim h$.

$Ra_L \sim L^3 \rightarrow h \sim L^{-1/4}$.

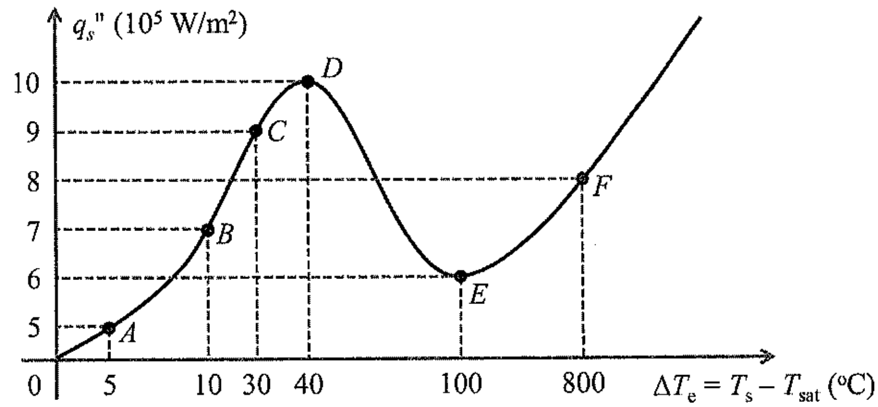
$L_A > L_B \rightarrow q_A < q_B$. The answer is (2).

Grading:

If the answer is correct, then give full points.

If the answer is wrong, then give partial credits based on the student's reasoning process.

(4) An electrical chip of a size 2x2 cm^2 is cooled by pool boiling of a dielectric fluid with a saturation temperature T_{sat} = 50°C under atmospheric pressure. The boiling curve of the fluid is given below. Then, the heat transfer coefficient under the critical heat flux condition is h = _25,000_ W/m^2-K. If the chip operates properly at temperatures up to 80°C, under the present cooling scheme the maximum heat dissipation rate would be _360_ W.



Solution:

At point D, $q_s'' = h_D \Delta T_e \Rightarrow h_D = \dfrac{q_s''}{\Delta T_e} = \dfrac{1000000}{40} = 25000$ W/m^2-K.

T_{max} = 80°C → ΔT_e = 30°C → q_s'' = 9 x 10^5 W/m^2 → q_s = (9 x 10^5) x (2 x 2 x 10^{-4}) = 360 W.

Grading:

If the answer is correct, then give full points.

If the answer is wrong, then give partial credits based on the student's reasoning process.

a) $u_\infty = 35 \ m/s$

$$Re = \frac{\rho u_\infty D}{\mu} = 1.65 \times 10^5$$

$$Pr = \frac{\nu}{\alpha} = \frac{\mu/\rho}{\alpha} = 0.706$$

$$\overline{Nu} = 2 + (0.4 Re^{\frac{1}{2}} + 0.06 Re^{\frac{2}{3}}) Pr^{.4} (\frac{\mu}{\mu_s})^{\frac{1}{4}} = 302$$

$$\bar{h} = \frac{\overline{Nu} \cdot k}{D} = \frac{\overline{Nu} \cdot \alpha \rho c_p}{D} = 106 \ W/m^2\text{-}K$$

b)

$$Sc = \frac{\nu}{D_{AB}} = \frac{\mu/\rho}{D_{AB}} = 0.611$$

$$\overline{Sh} = 2 + (0.4 Re^{\frac{1}{2}} + 0.06 Re^{\frac{2}{3}}) Sc^{0.4} (\frac{\mu}{\mu_s})^{\frac{1}{4}} = 285$$

$$\overline{h_m} = \overline{Sh} \cdot \frac{D_{AB}}{D} = 9.88 \times 10^{-2} \ m/s$$

↖ Alternatively $\frac{Sh}{Nu} = (\frac{Sc}{Pr})^n$ $(n = 0.4)$

$$\overline{n_A''} = \overline{h_m} \cdot (\rho_{As} - \rho_{A\infty}) = h_m \rho_{As}$$

$$pV = \frac{m}{M} RT$$

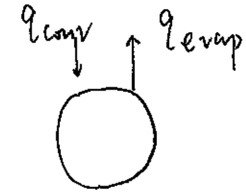
$$\rho = \frac{m}{V} = \frac{pM}{RT}$$

$$\rho_{As} = p_{sat} \cdot \frac{M_{H_2O}}{RT} = \frac{1920 \times 18 \times 10^{-3}}{8.314 \times 290} = 1.41 \times 10^{-3} \ kg/m^3$$

$$\therefore \overline{n_A''} = 1.41 \times 10^{-3} \ kg/m^2\text{-}s$$

c)

$$q_{evap} = \overline{n_A''} \cdot A \cdot h_{fg} = \overline{n_A''} \cdot 4\pi (\tfrac{D}{2})^2 \cdot h_{fg} = 61.3 \ W$$



$$q_{conv} = \bar{h} \cdot A \cdot \Delta T = \bar{h} \cdot 4\pi (\tfrac{D}{2})^2 \cdot \Delta T = 37.5 \ W$$

$q_{net} = q_{evap} - q_{conv} = 23.8 \ W$, out of the ball

a) For incompressible flow:

$$\dot{m} = \rho \cdot (A \cdot u_m)$$

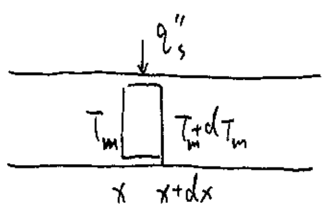
$$\therefore u_m = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho \cdot \frac{\pi}{4} D^2} = 3.18 \times 10^{-2} \quad m/s \quad = \frac{0.01}{1000 \times \frac{\pi}{4} \times 0.02^2}$$

b) $$Re = \frac{\rho V D}{\mu} = \frac{\rho u_m D}{\mu} = 1156 < 2300$$

$\therefore Nu = 4.36$ For fully developed region, for const q'' condition

$$h = Nu \cdot \frac{k}{D} = 142 \quad W/m^2\text{-}K$$

c)



$$\dot{m} C_p (T_m + dT_m) = \dot{m} C_p T_m + q_s'' \cdot dA$$

$$dA = P \cdot dx \quad , \quad P = \pi D$$

$$\therefore \dot{m} C_p \frac{dT_m}{dx} = q_s'' \cdot P$$

$$q_s'' = \frac{\dot{m} C_p \frac{dT_m}{dx}}{\pi D} = \frac{\dot{m} C_p \frac{dT_s}{dx}}{\pi D} = 668 \quad W/m^2$$

(h is const, $T_s - T_m$ is const)

d) $$q_s'' = h (T_s - T_m)$$

$$\therefore T_s - T_m = q_s''/h = 4.7 \,^\circ C$$

$$T_{s,o} = T_s \big|_{x=10} = 60 \,^\circ C$$

$$T_{m,o} = T_{s,o} - 4.7 = 55.3 \,^\circ C$$