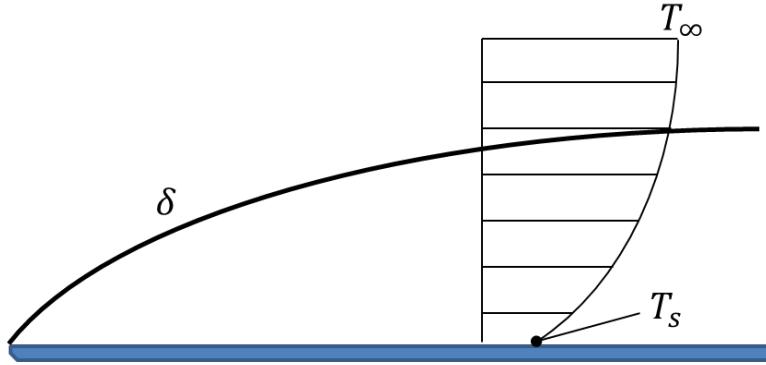


## Problem 1 (a)



### i) Answer: b. $\delta < \delta_t$

From the figure, it can be inferred that the thermal boundary layer,  $\delta_t$  is thicker than the momentum boundary layer,  $\delta$  based on the temperature profile provided.

### ii) Answer: a. From the fluid to the plate

From the temperature profile, it can be said that  $T_\infty > T_s$ . Therefore, the direction of heat transfer should be from the fluid to the plate.

### iii) Answer: a. $\delta > \delta_c$

The relationship between the momentum boundary layer thickness,  $\delta$  and the concentration boundary layer thickness,  $\delta_c$  is the following;

$$\frac{\delta}{\delta_c} = Sc^n ; n = \frac{1}{3},$$

where  $Sc$  is the Schmidt number. The Schmidt number is given as  $Sc = 10$ , therefore the following relationship can be drawn;

$$\frac{\delta}{\delta_c} = (10)^{\frac{1}{3}} = 2.15443 \rightarrow \delta = (2.15443) \cdot \delta_c,$$

$$\therefore \delta > \delta_c$$

### **Problem 1 (b)**

$$D_2 = 2 \cdot D_1 \quad \text{and} \quad V_1 = 2 \cdot V_2$$

$$\begin{aligned}\text{Re}_1 &= \frac{\rho V_1 D_1}{\mu} \\ \text{Re}_2 &= \frac{\rho V_2 D_2}{\mu} = \frac{\rho \left(\frac{1}{2}V_1\right)(2D_1)}{\mu} = \text{Re}_1\end{aligned}$$

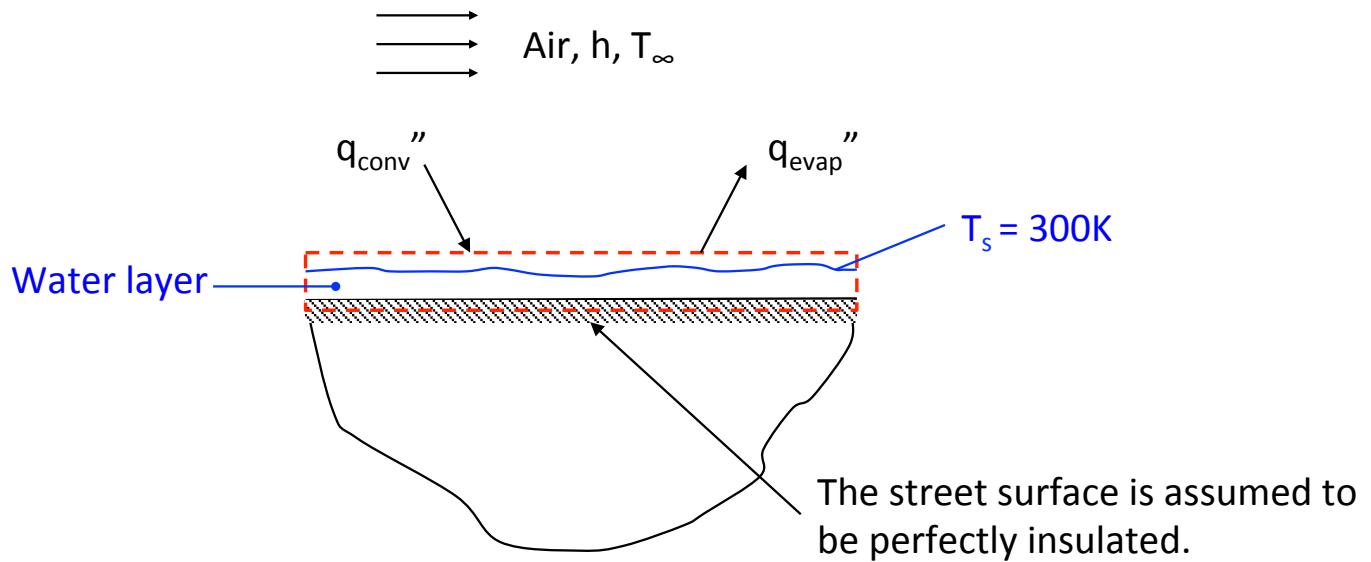
$$\text{Pr}_1 = \text{Pr}_2$$

$$Nu = \frac{h \cdot D}{k} = fn(\text{Re}, \text{Pr}) \rightarrow Nu_1 = Nu_2$$

$$h_1 = \frac{Nu_1 \cdot k}{D_1} \quad \text{and} \quad h_2 = \frac{Nu_2 \cdot k}{D_2} = \frac{Nu_1 \cdot k}{2 \cdot D_1} = \frac{1}{2} \cdot h_1$$

$$\boxed{\therefore h_1 = 2 \cdot h_2}$$

## Problem 2



(a)

### Assumptions

- 1) Steady state conditions
- 2) Constant properties
- 3) Radiation is negligible
- 4) The street surface is perfectly insulated
- 5) Heat-mass transfer analogy is applicable

$$\dot{E}_{in}'' - \dot{E}_{out}'' + \dot{E}_{gen}'' = \dot{E}_{store}'' ; \dot{E}_{gen}'' = \dot{E}_{store}'' = 0$$

$$\therefore \dot{E}_{in}'' = \dot{E}_{out}''$$

$$\dot{E}_{in}'' = q_{\text{conv}}'' = h(T_\infty - T_s)$$

$$\dot{E}_{out}'' = q_{\text{evap}}'' = n_A'' h_{fg} = h_m (\rho_{A,\text{sat at } T_s} - \rho_{A,\infty}) h_{fg} ; \quad \rho_{A,\infty} = 0 \text{ (dry air)}$$

$$\frac{Nu}{Pr^n} = \frac{Sh}{Sc^n} \rightarrow \frac{h}{h_m} = \rho C_p Le^{1-n} = \rho C_p \left( \frac{\alpha}{D_{AB}} \right)^{\frac{2}{3}}; \quad n = \frac{1}{3}$$

$$\dot{E}_{in}'' = \dot{E}_{out}'' \rightarrow h(T_\infty - T_s) = h_m \rho_{A, sat at T_s} h_{fg}$$

$$\rightarrow T_\infty = T_s + \frac{h_m}{h} \rho_{A, sat at T_s} h_{fg} = T_s + \frac{\rho_{A, sat at T_s} h_{fg}}{\rho C_p Le^{2/3}}$$

$$\boxed{\therefore T_\infty = T_s + \frac{h_m}{h} \rho_{A, sat at T_s} h_{fg} = T_s + \frac{\rho_{A, sat at T_s} h_{fg}}{\rho C_p Le^{2/3}}}$$

(b)

Evaluate water properties at  $T_s = 300$  K;

$$\rho_{A,s} = \frac{1}{v_g} = \frac{1}{39.13 m^3 / kg} = 0.025556 \text{ kg/m}^3$$

$$h_{fg} = 2438 \times 10^3 \text{ J/kg}$$

$$T_\infty = T_s + \frac{\rho_{A, sat at T_s} h_{fg}}{\rho C_p Le^{2/3}}$$

$$= 300 k + \frac{(0.025556 \text{ kg/m}^3) \cdot (2438 \times 10^3 \text{ J/kg})}{(1.161 \text{ kg/m}^3) \cdot (1007 \text{ J/kg} - K) \cdot \left( \frac{22.5 \times 10^{-6} \text{ m}^2/\text{s}}{26 \times 10^{-6} \text{ m}^2/\text{s}} \right)^{2/3}} = 358.68 \text{ K}$$

$$\boxed{\therefore T_\infty = 358.68 \text{ K}}$$

### Problem 3

(a)

$$Bi = \frac{h L_c}{k_{solid}} = \frac{h (r_0 / 3)}{k_{solid}} = \frac{(100 \text{ W} / \text{m}^2 - K) \cdot (0.1 \text{ m} / 3)}{350 \text{ W} / \text{m} - K} = 0.009524 << 0.1$$

The lump capacitance model is VALID.

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp[-(Bi) \cdot (Fo)] = \exp\left[-(Bi) \cdot \left(\frac{\alpha t}{L_c^2}\right)\right]$$

$$\frac{500 \text{ K} - 300 \text{ K}}{800 \text{ K} - 300 \text{ K}} = \exp\left[-(0.009524) \cdot \frac{(2.5 \times 10^{-6} \text{ m}^2 / \text{s}) \cdot t}{(0.1 \text{ m} / 3)^2}\right]$$

$$\boxed{\therefore t = 42,759.4 \text{ s}}$$

(b)

$$Bi = \frac{h L_c}{k_{solid}} = \frac{h (r_0 / 3)}{k_{solid}} = \frac{(100 \text{ W} / \text{m}^2 - K) \cdot (0.1 \text{ m} / 3)}{1.0 \text{ W} / \text{m} - K} = 3.33 >> 0.1$$

The lump capacitance model is NOT VALID, and the analytical solution for a sphere needs be used .

$$\theta_0^* = \frac{T_{center} - T_\infty}{T_i - T_\infty} = C_1 \cdot \exp(-\zeta_1^2 \cdot Fo); \quad Fo = \frac{\alpha t}{L_c^2} \quad \text{where} \quad L_c = r_0$$

$$Bi = \frac{h L_c}{k_{solid}} = \frac{h r_0}{k_{solid}} = \frac{(100 \text{ W} / \text{m}^2 - K) \cdot (0.1 \text{ m})}{1.0 \text{ W} / \text{m} - K} = 10$$

From table 5.1 using the Bi = 10, C<sub>1</sub> = 1.9249 and ζ<sub>1</sub> = 2.8363.

$$\theta_0^* = \frac{T_{center} - T_\infty}{T_i - T_\infty} = \frac{500 \text{ K} - 300 \text{ K}}{800 \text{ K} - 300 \text{ K}} = (1.9249) \cdot \exp \left[ - (2.8363)^2 \cdot \frac{(2.5 \times 10^{-6} \text{ m}^2 / \text{s}) \cdot t}{(0.1 \text{ m})^2} \right]$$

$$\therefore t = 781.23 \text{ s}$$