<Solution for Problem 1>

(a)

+2 $Air \ \ at \ \ 350K \ \Rightarrow \nu_{air} = 20.92 \times 10^{-6} \ \ m^2/s, \ \Pr_{air} = 0.7, \ k_{air} = 30 \times 10^{-3} \ \ W/m-K$

+3 $$\mathrm{Re}_D = \frac{V \cdot D}{\nu} = \frac{4 \ \ m/s \times 0.04 \ \ m}{20.92 \times 10^{-6} \ \ m^2/s} = 7646.49$$

+3 $$\overline{Nu_D} = 0.3 + \frac{0.62 \ \mathrm{Re}_D^{\frac{1}{2}} \ \Pr^{\frac{1}{3}}}{\left[1 + (0.4 \ / \ \Pr)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \cdot \left[1 + \left(\frac{\mathrm{Re}_D}{28{,}2000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}} = 46.04$$

+2 $$\bar{h} = \frac{k}{D} \cdot \overline{Nu_D} = \frac{30 \times 10^{-3} \ \ W/m-K}{0.04 \ \ m} \cdot (46.04) = 34.53 \ \ W/m^2-K$$

$$\boxed{\therefore \ \ \bar{h} = 34.53 \ \ W/m^2-K}$$

(b)

+4 $$Bi = \frac{h \cdot L_c}{k_g} = \frac{h \cdot R}{k_g} = \frac{(34.53 \ \ W/m^2-K) \cdot (0.02 \ \ m)}{1.38 \ \ W/m-K} = 0.5$$

$For \ \ Bi = 0.5, \ \varsigma_1 = 0.9408 \ \ and \ \ C_1 = 1.1143 \ (cylinder)$

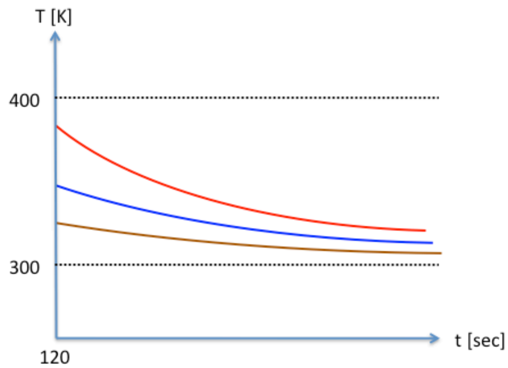
+3 $$Fo = \frac{\alpha \cdot t}{R^2} = \frac{(0.8 \times 10^{-6} \ \ m^2/s) \cdot (120 \ \ s)}{(0.02 \ \ m)^2} = 0.24 > 0.2$$

+3 $$\theta_0^* = \frac{T_{r=0,t=120s} - T_\infty}{T_i - T_\infty} = C_1 \cdot \exp\left(-\varsigma_1^2 \cdot Fo\right) = 1.1143 \cdot \exp\left(-0.9408^2 \cdot 0.24\right) = 0.901046$$

+2 $$T_{r=0,t=120s} = T_\infty + \theta_0^* \cdot (T_i - T_\infty) = 300 \ \ K + 90.1 \ \ K = 390.1 \ \ K$$

$$\boxed{\therefore \ \ T_{r=0,t=120s} = 390.1 \ \ K}$$

(c) +8



(red line)	TC1 (r = 0)	exponential
(blue line)	TC2 (r = R/2)	exponential
(brown line)	TC3 (r = R)	exponential

No Assumptions -1

<Solution for Problem 2>

(a)

+3 $q_{electric} = q_{conv} + q_{evap}$

+4 $q_{conv} = h \cdot A \cdot (T_s - T_\infty)$

+4 $q_{evap} = \dot{m} \cdot h_{fg} = n_A''[\rho_{A,sat}(T_s) - \rho_{A,\infty}(T_\infty)] \cdot A \cdot h_{fg}$

$where \ \dot{m} = n_A'' \cdot A = h_m \cdot A \cdot [\rho_{A,sat}(T_s) - \rho_{A,\infty}(T_\infty)]$

Overall,

$$q_{electric} = \frac{\dot{m} \cdot (T_s - T_\infty)}{\rho_{A,sat}(T_s) - \rho_{A,\infty}(T_\infty)} \cdot \rho c_p Le^{2/3} + \dot{m} \cdot h_{fg}$$

+1 $$\boxed{\therefore \ q_{electric} = q_{conv} + q_{evap} \ \ or \ \ q_{electric} = \frac{\dot{m} \cdot (T_s - T_\infty)}{\rho_{A,sat}(T_s) - \rho_{A,\infty}(T_\infty)} \cdot \rho c_p Le^{2/3} + \dot{m} \cdot h_{fg}}$$

(b)

+4 $$\frac{h}{h_m} = \frac{h \cdot A}{h_m \cdot A} = \rho c_p Le^{2/3}$$

$$h \cdot A = h_m \cdot A \cdot \rho c_p Le^{2/3} = \frac{\dot{m}}{\rho_{A,sat}(T_s) - \rho_{A,\infty}(T_\infty)} \cdot \rho c_p Le^{2/3}$$

+4 $where \ Le = \dfrac{\alpha}{D_{AB}}$

+4 $$\boxed{\therefore \ h \cdot A = \frac{\dot{m}}{\rho_{A,sat}(T_s) - \rho_{A,\infty}(T_\infty)} \cdot \rho c_p Le^{2/3}}$$

(c)

+1 $T_{film} = \frac{T_s + T_\infty}{2} = \frac{(310 + 290)\ K}{2} = 300\ K$

$At\ 300\ K,$

+1 $\rho_{air} = 1.1614 kg/m^3,\ c_{p,air} = 1{,}007\ J/kg-K,\ \alpha_{air} = 22.5 \times 10^{-6}\ m^2/s,\ D_{AB} = 26 \times 10^{-6}\ m^2/s$

$At\ T_s = 310\ K,$

+1 $\rho_{A,s} = \frac{1}{v_{g,air}} = \frac{1}{22.93\ m^3/kg} = 0.043611\ kg/m^3,\ h_{fg} = 2{,}414 \times 10^3\ J/kg$

+1 $Also,\ \rho_{A,\infty} = 0\ (dry\ air)$

+1 $q_{electric} = \frac{\dot{m} \cdot (T_s - T_\infty)}{\rho_{A,sat}(T_s) - \rho_{A,\infty}(T_\infty)} \cdot \rho_{air} c_{p,air} Le^{2/3} + \dot{m} \cdot h_{fg} = \frac{\dot{m} \cdot (T_s - T_\infty)}{\rho_{A,sat}(T_s)} \cdot \rho_{air} c_{p,air} \left(\frac{\alpha}{D_{AB}} \right)^{2/3} + \dot{m} \cdot h_{fg}$

$$\therefore\ q_{electric} = \frac{\dot{m} \cdot (310-290)K}{0.043611\ kg/m^3} \cdot \left(1.1614 kg/m^3\right) \cdot \left(1{,}007 J/kg-K\right) \left(\frac{22.5 \times 10^{-6}\ m^2/s}{26 \times 10^{-6}\ m^2/s} \right)^{2/3} + \dot{m} \cdot \left(2{,}414 \times 10^3\ J/kg\right) = 3{,}000\ W$$

$\Rightarrow\ \dot{m} = 0.001034\ kg/s$

+1 $\boxed{\therefore\ \dot{m} = 0.001034\ kg/s}$

No Assumptions -1

<Solution for Problem 3>

(a)

$At\ \ T_{film,i} = 162.5°C,\ v_{air} = 30.67\times10^{-6}\ \ m^2/s,\ \Pr_{air} = 0.687,\ k_{air} = 0.0363\ \ W/m-K$

+3 $\mathrm{Re}_L = \dfrac{U_\infty L}{v} = \dfrac{(30\ \ m/s)\cdot(1\ \ m)}{30.67\times10^{-6}\ \ m^2/s} = 9.781155\times10^5 > \mathrm{Re}_{critical} = 5\times10^5 \Rightarrow mixed\ \ boundary\ \ layer$

+3

$$\therefore\ \mathrm{Re}_L = 9.781155\times10^5 \Rightarrow mixed\ \ boundary\ \ layer$$

(b)

+4 $\overline{Nu_L} = \left(0.037\cdot\mathrm{Re}_L{}^{4/5}-871\right)\cdot\Pr^{1/3} = \left[0.037\cdot\left(9.78155\times10^5\right)^{4/5}-871\right]\cdot(0.687)^{1/3} = 1{,}255.31$

+4 $\overline{Nu_L} = \dfrac{\overline{h_L}L_c}{k_{air}} = 1{,}255.31 \Rightarrow \overline{h_L} = 45.5678\ \ W/m^2-K$

$$\therefore\ \overline{h_L} = 45.5678\ \ W/m^2-K$$

(c)

+3 $Bi = \dfrac{\overline{h_L}\cdot L_c}{k_s}\ \ where\ \ L_c = \dfrac{V}{A_s} = \dfrac{D}{4}$

+2 $Bi = \dfrac{\left(45.5678\ \ W/m^2-K\right)\cdot\left(\dfrac{0.1\ \ m}{4}\right)}{100\ \ W/m-K} = 0.011392 < 0.1 \Rightarrow Lump\ \ capacitance\ \ analysis\ \ is\ \ valid.$

+3

$$\therefore Bi = 0.011392 \Rightarrow Lump\ \ capacitance\ \ analysis\ \ is\ \ valid.$$

(d)

$$\underline{+4} \quad \frac{\theta}{\theta_i} = \frac{T - T_i}{T_\infty - T_i} = \frac{(30 - 25)°C}{(300 - 25)°C} = 0.018182 = \exp(-Bi \cdot Fo)$$

$$where \quad Fo = \frac{\alpha \cdot t}{L_c^{\,2}} \quad and \quad \alpha = \frac{k_s}{\rho_s \cdot c_{p,s}} = \frac{100 \;\; W/m-k}{\left(2{,}702 \;\; kg/m^3\right)\cdot\left(903 \;\; J/kg-K\right)} = 4.0985 \times 10^{-5} \;\; m^2/s$$

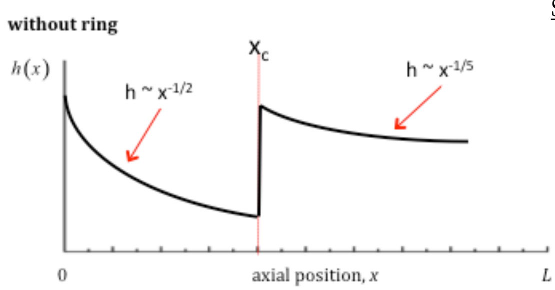
$$\underline{+2} \quad \therefore \frac{\theta}{\theta_i} = 0.018182 = \exp\left[-(0.011392)\cdot\frac{\left(4.0985 \times 10^{-5} \;\; m^2/s\right)\cdot t_{30°C}}{\left(\dfrac{0.1 \;\; m}{4}\right)^2}\right]$$

$$\underline{+4} \quad Solve \;\; for \;\; t_{30°C} \;\; \Rightarrow \;\; t_{30°C} = 5{,}364.28 \;\; seconds$$

$$\boxed{\therefore t_{30°C} = 5{,}364.28 \;\; seconds = 1.49 \;\; hrs.}$$

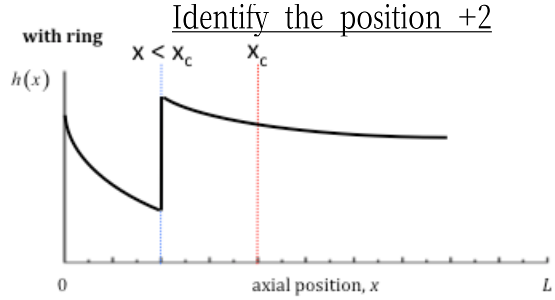
(e)

$Re_{x=10cm} < Re_{x,critical} \Rightarrow$ The turbulent transition is tripped earlier than without the ring case



showing this +5

Identify the position +2



For laminar flow, h ~ $x^{-1/2}$, while for turbulent flow h ~ $x^{-1/5}$.

- Hence h varies more slowly for turbulent flow, and also has larger magnitude at given location, x.
- Thus, by having a larger portion of the surface as turbulent (*e.g.* by tripping the boundary layer at $Re_x < Re_{x,c}$), $\overline{h_L}$ will be larger.

Correct answer (greater heat transfer) +1