# ME 315
# Exam 2
# 8:00 -9:00 PM
# Tuesday, March 10, 2009

- This is a closed-book, closed-notes examination. *There is a formula sheet at the back.*
- You must turn off all communications devices before starting this exam, and leave them off for the entire exam.
- Please write legibly and show all work for your own benefit. Please show your final answers in the boxes provided.
- State all assumptions.
- Please arrange all your sheets in the correct order. Make sure they are all included.

**Name: ___________________________________**
**Last                                        First**

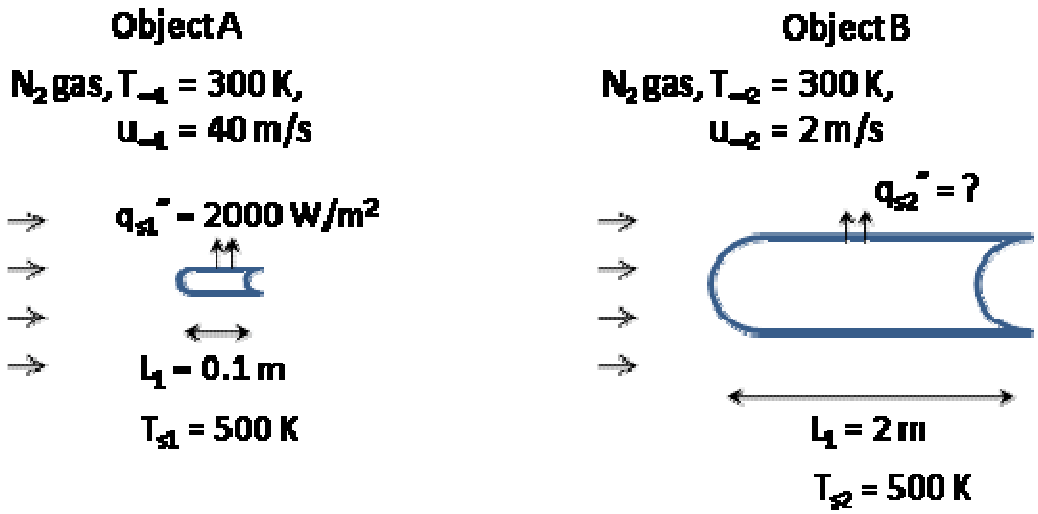
## CIRCLE YOUR DIVISION

**Div. 1 (9:30 am)**
**Prof. Murthy**

**Div. 2 (12:30 pm)**
**Prof. Choi**

| Problem | Score |
|---|---|
| 1 (20 Points) | |
| 2 (40 Points) | |
| 3 (40 Points) | |
| Total (100 Points) | |

1. Consider the following two objects with the same geometry but different sizes exposed to nitrogen gas. The two objects are at 500 K, while the $N_2$ gas has a temperature of 300 K.




All properties may be evaluated at 400 K: For nitrogen, you are given thermal conductivity, $k = 26.0 \times 10^{-3}$ W/mK; density, $\rho = 1.0$ kg/m$^3$; specific heat, $c_p = 1000$ J/kgK; kinematic viscosity, $\nu = 18.2 \times 10^{-6}$ m$^2$/s.

With given conditions, the heat flux in object A is found to be $q_{s1}'' = 2{,}000$ W/m$^2$. What is the average convective heat transfer coefficient ($h_2$) and heat flux in $q_{s2}''$ in object B?

$h_2 =$ W/m$^2$K

$q_{s2}'' =$ W/m$^2$

**Solution**

$$\overline{Nu} = f(Re, Pr) = \frac{h\,L_c}{k}\;;\;\; Re = \frac{\rho U\,L_c}{\mu} \quad Pr = \frac{\nu}{\alpha}$$

$$\alpha_1 = \alpha_2 \rightarrow Pr_1 = Pr_2$$

$$20\cdot L_{c1} = L_{c2}, U_1 = 20\cdot U_2 \rightarrow Re_1 = Re_2$$

$$\therefore \overline{Nu}_1 = \overline{Nu}_2 \rightarrow \frac{h_1\,L_{c1}}{k} = \frac{h_2\,L_{c2}}{k}$$

$$\therefore h_2 = \frac{h_1\,L_{c1}}{L_{c2}} = \frac{h_1\,L_{c1}}{20\cdot L_{c1}} = \frac{h_1}{20}$$

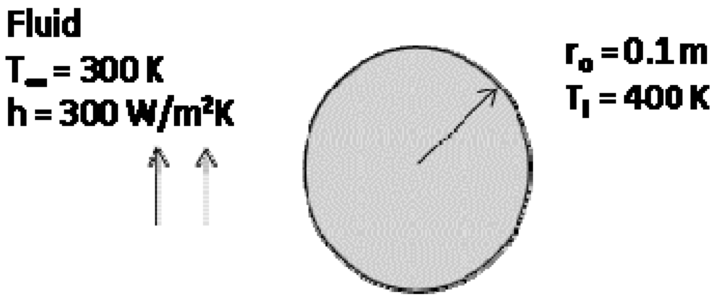
$$q_1'' = h_1\cdot(T_{s,1} - T_\infty) = h_1\cdot(500-300) = 2000\,W/m^2 \rightarrow h_1 = 10\,W/m^2-K$$

$$\boxed{\therefore h_2 = \frac{h_1}{20} = 0.5\,W/m^2-K}$$

$$q_2'' = h_2\cdot(T_{s,2} - T_\infty) = (0.5\,W/m^2-K)\cdot(500-300)K = 100\,W/m^2$$

$$\boxed{\therefore q_2'' = 100\,W/m^2}$$

2. A spherical metal ball of a radius, $r_o = 0.1$ m is initially at $T_i = 400$ K. At t = 0, the ball is submerged in a fluid, where convective heat transfer coefficient, h, is 300 W/m$^2$K and the temperature, $T_\infty$, is 300 K. This ball is assumed to have a uniform temperature at any time.




You are given the following properties of the ball: Thermal conductivity, $k = 30$ W/mK; density, $\rho = 9000$ kg/m$^3$; heat capacity, $c_p = 500$ J/kgK.

(i) Determine thermal time constant ($\tau_t$) of the ball and the temperature of the ball at $t = \tau_t$.

| | |
|---|---|
| $\tau_t$ = | second |
| T = | K |

(ii) Evaluate the validity of the uniform temperature assumption. Provide reasoning.

| | | |
|---|---|---|
| Valid ? | Invalid ? | (Circle one.) |
| Reason: | | |

If your answer states Valid, stop here and proceed to problem #3. If it is Invalid, go to (iii) on the next page.

(iii) Determine the temperature at the center of the ball at t = $\tau_t$.

| |
|---|
| T = K |

**Solution**

(i)

$$\tau_t = \frac{\rho V C_p}{h_{conv} A_s}$$

$$V = \frac{4}{3}\pi r_o^3$$

$$A_s = 4\pi r_o^2$$

$$\therefore\ \tau_t = \frac{\rho V C_p}{h_{conv} A_s} = \frac{\rho \frac{4}{3}\pi r_o^3 C_p}{h_{conv} 4\pi r_o^2} = \frac{\rho r_o C_p}{3 h_{conv}} = \frac{(9000\,kg/m^3)\cdot(0.1m)\cdot(500\,J/kg-K)}{3\cdot(300\,W/m^2-K)} = 500\sec$$

$$\boxed{\therefore\ \tau_t = 500\sec}$$

$$\frac{\theta}{\theta_i} = \frac{T-T_\infty}{T_i-T_\infty} = exp\,(-\frac{t}{\tau_t})\ ;\ t=\tau_t \quad \therefore \frac{t}{\tau_t}=1$$

$$\frac{T-T_\infty}{T_i-T_\infty} = exp\,(-1)$$

$$\therefore T = T_\infty + (T_i - T_\infty)\cdot exp\,(-1) = 300 + (400-300)\cdot exp\,(-1) = 336.788K$$

$$\boxed{\therefore T_{t=\tau_t} = 336.788K}$$

(ii)

$$Bi = \frac{h_{conv} \cdot L_c}{k_{solid}} \quad ; \quad L_c = \frac{r_o}{3} = \frac{0.1\,m}{3}$$

$$\therefore Bi = \frac{(300\,W/m^2-K)\cdot(\frac{0.1\,m}{3})}{(30\,W/m-K)} = 0.333 > 0.1$$

$$\boxed{\therefore Invalid, beacuse\, Bi = 0.333 > 0.1}$$

(iii)

$$Bi = \frac{h_{conv} \cdot r_o}{k_{solid}} = \frac{(300\,W/m^2-K)\cdot(0.1\,m)}{30\,W/m-K} = 1.0$$

*From the Table* 5.1,

$$\zeta_1 = 1.5708, \quad C_1 = 1.2732$$

$$\theta_o^* = \frac{T-T_\infty}{T_i-T_\infty} = C_1 \cdot exp\,(-\zeta_1 \cdot Fo)$$

$$Fo = \frac{\alpha\, t}{r_o^2} \quad ; \quad \alpha = \frac{k}{\rho \cdot C_p}$$

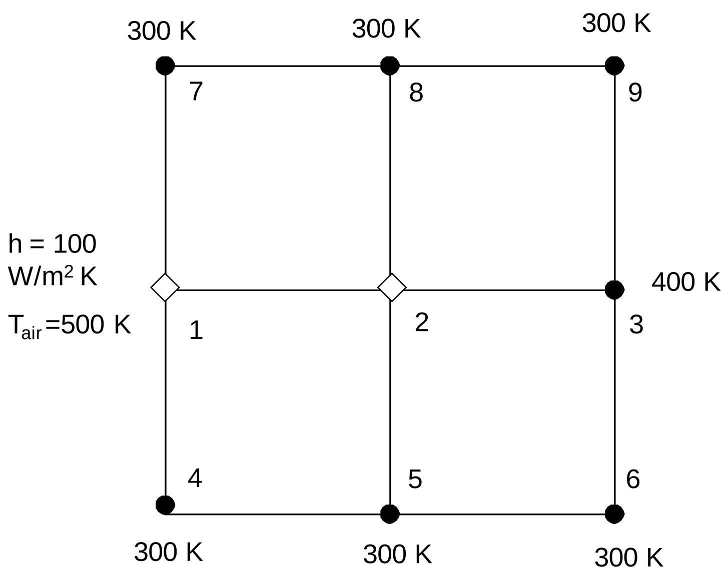
$$\therefore Fo = \frac{k}{\rho \cdot C_p}\cdot\frac{t}{r_o^2} = \frac{(30\,W/m-K)}{(9000\,kg/m^3)\cdot(500\,J/kg-K)}\cdot\frac{(500\text{ sec})}{(0.1\,m)^2} = 0.333$$

$$\therefore \theta_o^* = \frac{T-T_\infty}{T_i-T_\infty} = \frac{T-300}{400-300} = (1.2732)\cdot exp\,[-(1.5708)^2\cdot(0.333)] = 0.5598$$

$$\boxed{\therefore T_{r=0,500\text{sec}} = 355.983\,K}$$

3. Consider unsteady heat conduction in a 2D square domain of side 2 m, as shown below. The domain is meshed with a square mesh with $\Delta x = \Delta y = 1$ m. The initial temperature at all grid points is 300 K. At time t=0, the boundary conditions shown below are applied. You are asked to determine how the temperatures of grid points 1 and 2 change with time using an explicit time stepping scheme.

You are given the following material properties:
$\rho = 2000$ kg/m$^3$  k=100 W/mK  $C_p = 300$ J/kgK




(i) You are given the choice of two time steps, $\Delta t = 500$ s and $\Delta t = 2000$ s. Which one would you choose, and why?

| |
|---|
| $\Delta t =$ seconds<br><br>Reason: |

(ii) Develop analytical equations for $T_1$ and $T_2$ at time step $t = \Delta t$ in terms of neighbor temperatures, material properties and mesh parameters. Show analytical expressions for the discrete equations here.

| |
|---|
| $T_1 =$<br><br><br>$T_2 =$ |

(iii) Using the value of Δt you chose in part (i), determine the numerical values of the coefficients in the discrete equations derived in part (ii).

| |
|---|
| $T_1$ = |
| $T_2$ = |

(iv) Find the numerical values of the temperatures $T_1$ and $T_2$ at t= Δt using the explicit scheme.

| |
|---|
| $T_1$ (t=Δt) = |
| $T_2$ (t=Δt) = |

**Solution**

(i)

$$For\ 2-D, the\ stability\ limit\ is\ Fo \le \frac{1}{4}$$

$$Fo = \frac{\alpha \cdot \Delta t}{(\Delta x)^2}\ ; \alpha = \frac{k}{\rho \cdot c_p} = \frac{100\ W/m-K}{(2000\ kg/m^3)\cdot(300\ J/kg-K)} = 0.000167\ m^2/s$$

$$\therefore Fo = \frac{(0.000167\ m^2/s)\cdot \Delta t}{(1\ m)^2} \le \frac{1}{4} \quad \rightarrow \quad \Delta t \le 1500\ s$$

$$\boxed{\therefore \Delta t = 500\ sec\ is\ suitable.}$$

**(ii)**

**Node 1**

$$\dot{E}_{in} - \dot{E}_{out} - \dot{E}_{gen} = \dot{E}_{st} \quad ; \quad \dot{E}_{in} = q'_E + q'_W + q'_S + q'_N \qquad \dot{E}_{out} = 0 \qquad \dot{E}_{gen} = 0$$

$$q'_E = -k\,\frac{\Delta y}{\Delta x}\,(T_1^p - T_2^p) \qquad\qquad q'_W = h\,\Delta y\,(T_{air} - T_1^p)$$

$$q'_S = -k\,\frac{\Delta x}{2\cdot\Delta y}\,(T_1^p - T_4^p) \qquad\qquad q'_N = -k\,\frac{\Delta x}{2\cdot\Delta y}\,(T_1^p - T_7^p)$$

$$\dot{E}_{st} = \rho\cdot c_p\cdot\frac{\Delta x}{2}\cdot\Delta y\cdot\frac{T_1^{p+1} - T_1^p}{\Delta t}$$

$$\therefore q'_E + q'_W + q'_S + q'_N = \dot{E}_{st}$$

$$k\,\frac{\Delta y}{\Delta x}\,(T_2^p - T_1^p) + h\,\Delta y\,(T_{air} - T_1^p) + k\,\frac{\Delta x}{2\cdot\Delta y}\,(T_4^p - T_1^p) + k\,\frac{\Delta x}{2\cdot\Delta y}\,(T_7^p - T_1^p) = \rho\cdot c_p\cdot\frac{\Delta x}{2}\cdot\Delta y\cdot\frac{T_1^{p+1} - T_1^p}{\Delta t}$$

$$k\,(T_2^p - T_1^p) + h\,\Delta y\,(T_{air} - T_1^p) + \frac{k}{2}\,(T_4^p - T_1^p) + \frac{k}{2}\,(T_7^p - T_1^p) = \rho\cdot c_p\cdot\frac{\Delta x}{2}\cdot\Delta y\cdot\frac{T_1^{p+1} - T_1^p}{\Delta t}$$

$$T_1^{p+1} = T_1^p + \frac{2\cdot\Delta t}{\rho\cdot c_p\cdot\Delta x\cdot\Delta y}\cdot[k\,(T_2^p - T_1^p) + h\,\Delta y\,(T_{air} - T_1^p) + \frac{k}{2}\,(T_4^p - T_1^p) + \frac{k}{2}\,(T_7^p - T_1^p)]$$

$$= T_1^p + 2\cdot Fo\,(T_2^p - T_1^p) + \frac{2\cdot h\cdot\Delta t}{\rho\cdot c_p\cdot\Delta x}(T_{air} - T_1^p) + Fo\cdot(T_4^p - T_1^p) + Fo\cdot(T_7^p - T_1^p)$$

$$(\because Fo = \frac{\alpha\cdot\Delta t}{\Delta x\cdot\Delta y} = \frac{\alpha\cdot\Delta t}{(\Delta x)^2} \quad and \quad \alpha = \frac{k}{\rho\cdot c_p})$$

$$\therefore T_1^{p+1} = T_1^p + \frac{2\cdot\Delta t}{\rho\cdot c_p\cdot\Delta x\cdot\Delta y}\cdot[k\,(T_2^p - T_1^p) + h\,\Delta y\,(T_{air} - T_1^p) + \frac{k}{2}\,(T_4^p - T_1^p) + \frac{k}{2}\,(T_7^p - T_1^p)]$$

*or*

$$T_1^{p+1} = T_1^p + 2\cdot Fo(T_2^p - T_1^p) + \frac{2\cdot h\cdot\Delta t}{\rho\cdot c_p\cdot\Delta x}(T_{air} - T_1^p) + Fo\cdot(T_4^p - T_1^p) + Fo\cdot(T_7^p - T_1^p)$$

**Node 2**

$$T_2^{p+1} = (1 - 4 \cdot Fo) \cdot T_2^p + Fo \cdot (T_1^p + T_3^p + T_5^p + T_8^p)$$

$$\therefore T_2^{p+1} = (1 - 4 \cdot Fo) \cdot T_2^p + Fo \cdot (T_1^p + T_3^p + T_5^p + T_8^p)$$

**(iii)**

$For\ \Delta t = 500\ sec,\ Fo = 0.083$

$$\frac{\alpha \cdot \Delta t}{\Delta x \cdot \Delta y} = \frac{\alpha \cdot \Delta t}{(\Delta x)^2} = Fo = 0.083 \quad ; \quad \Delta x = \Delta y$$

$$\frac{2 \cdot h\ \Delta t}{\rho \cdot c_p \cdot \Delta x} = \frac{2 \cdot (100\ W/m^2 - K) \cdot (500\ sec)}{(2000\ kg/m^3) \cdot (300\ J/kg - K) \cdot (1\ m)} = 0.1667$$

$$1 - 4 \cdot Fo = 1 - 4 \cdot 0.083 = 0.6667$$

$$T_1^{p+1} = T_1^p + 2 \cdot Fo\,(T_2^p - T_1^p) + \frac{2 \cdot h \cdot \Delta t}{\rho \cdot c_p \cdot \Delta x}(T_{air} - T_1^p) + Fo \cdot (T_4^p - T_1^p) + Fo \cdot (T_7^p - T_1^p)$$
$$= T_1^p + 2 \cdot (0.083) \cdot (T_2^p - T_1^p) + (0.1667) \cdot (T_{air} - T_1^p) + (0.083) \cdot (T_4^p - T_1^p) + (0.083) \cdot (T_7^p - T_1^p)$$

$$T_2^{p+1} = (1 - 4 \cdot Fo) \cdot T_2^p + Fo \cdot (T_1^p + T_3^p + T_5^p + T_8^p)$$
$$= [1 - 4 \cdot (0.083)] \cdot T_2^p + (0.083) \cdot (T_1^p + T_3^p + T_5^p + T_8^p)$$
$$= (0.6667) \cdot T_2^p + (0.083) \cdot (T_1^p + T_3^p + T_5^p + T_8^p)$$

$$\therefore T_1^{p+1} = T_1^p + 2 \cdot (0.083) \cdot (T_2^p - T_1^p) + (0.1667) \cdot (T_{air} - T_1^p) + (0.083) \cdot (T_4^p - T_1^p) + (0.083) \cdot (T_7^p - T_1^p)$$

$$T_2^{p+1} = (0.6667) \cdot T_2^p + (0.083) \cdot (T_1^p + T_3^p + T_5^p + T_8^p)$$

**(iv)**

$$T_1^{p+1} = T_1^p + 2\cdot(0.083)\cdot(T_2^p - T_1^p) + (0.1667)\cdot(T_{air} - T_1^p) + (0.083)\cdot(T_4^p - T_1^p) + (0.083)\cdot(T_7^p - T_1^p)$$
$$= 300 + 2\cdot(0.083)\cdot(300 - 300) + (0.1667)\cdot(500 - 300) + (0.083)\cdot(300 - 300) + (0.083)\cdot(300 - 300)$$
$$= 333.34\ K$$

$$T_2^{p+1} = (0.6667)\cdot T_2^p + (0.083)\cdot(T_1^p + T_3^p + T_5^p + T_8^p)$$
$$= (0.6667)\cdot(300) + (0.083)\cdot(300 + 400 + 300 + 300)$$
$$= 307.91\ K$$

$$\therefore T_1^{p+1} = 333.34\ K \qquad T_2^{p+1} = 307.91\ K$$