ME 315
Exam 2
8:00 -9:00 PM
Tuesday, March 10, 2009

- This is a closed-book, closed-notes examination. *There is a formula sheet at the back.*
- You must turn off all communications devices before starting this exam, and leave them off for the entire exam.
- Please write legibly and show all work for your own benefit. Please show your final answers in the boxes provided.
- State all assumptions.
- Please arrange all your sheets in the correct order. Make sure they are all included.

Name: ___________________________________
Last   First

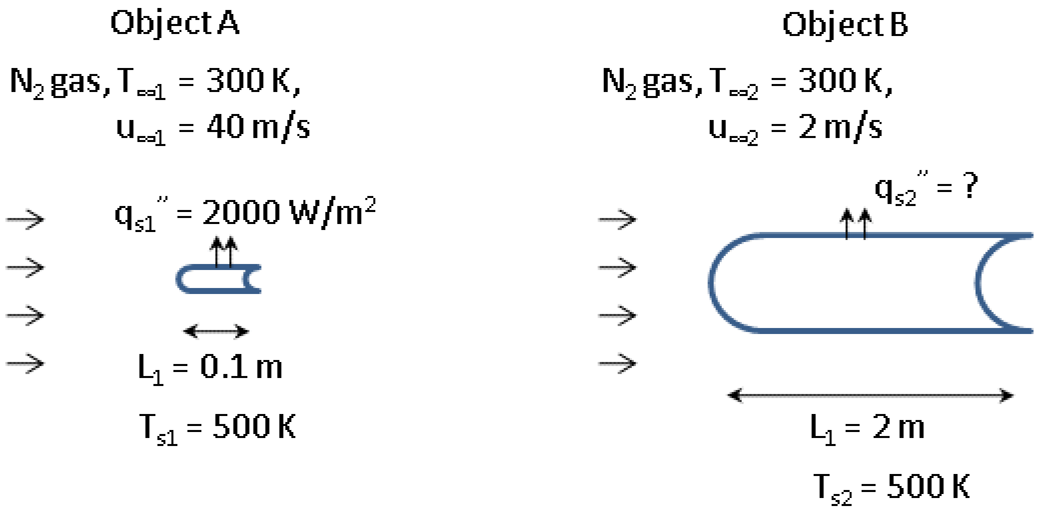
CIRCLE YOUR DIVISION

Div. 1 (9:30 am)
Prof. Murthy

Div. 2 (12:30 pm)
Prof. Choi

Problem	Score
1 (20 Points)	
2 (40 Points)	
3 (40 Points)	
Total (100 Points)	

1. Consider the following two objects with the same geometry but different sizes exposed to nitrogen gas. The two objects are at 500 K, while the N_2 gas has a temperature of 300 K.



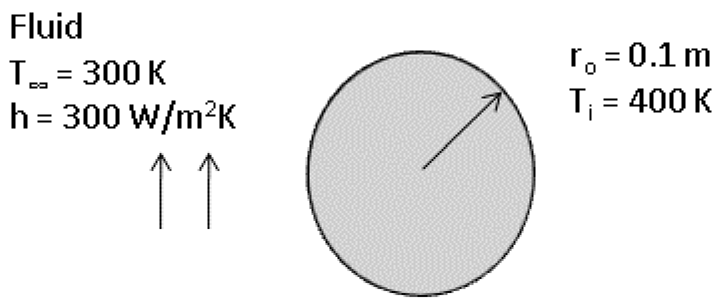
All properties may be evaluated at 400 K: For nitrogen, you are given thermal conductivity, k = 26.0 × 10^{-3} W/mK; density, ρ = 1.0 kg/m^3; specific heat, c_p = 1000 J/kgK; kinematic viscosity, ν = 18.2 × 10^{-6} m^2/s.

With given conditions, the heat flux in object A is found to be q_{s1}'' = 2,000 W/m^2. What is the average convective heat transfer coefficient (h_2) and heat flux in q_{s2}'' in object B?

h_2 = W/m^{2}K

q_{s2}''= W/m^2

2. A spherical metal ball of a radius, r_o = 0.1 m is initially at T_i = 400 K. At t = 0, the ball is submerged in a fluid, where convective heat transfer coefficient, h, is 300 W/m^2K and the temperature, T_∞, is 300 K. This ball is assumed to have a uniform temperature at any time.



You are given the following properties of the ball: Thermal conductivity, k = 30 W/mK; density, ρ = 9000 kg/m^3; heat capacity, c_p = 500 J/kgK.

(i) Determine thermal time constant (τ_t) of the ball and the temperature of the ball at t = τ_t.

τ_t =	second
T =	K

(ii) Evaluate the validity of the uniform temperature assumption. Provide reasoning.

Valid ?	Invalid ?	(Circle one.)
Reason:		

If your answer states Valid, stop here and proceed to problem #3. If it is Invalid, go to (iii) on the next page.

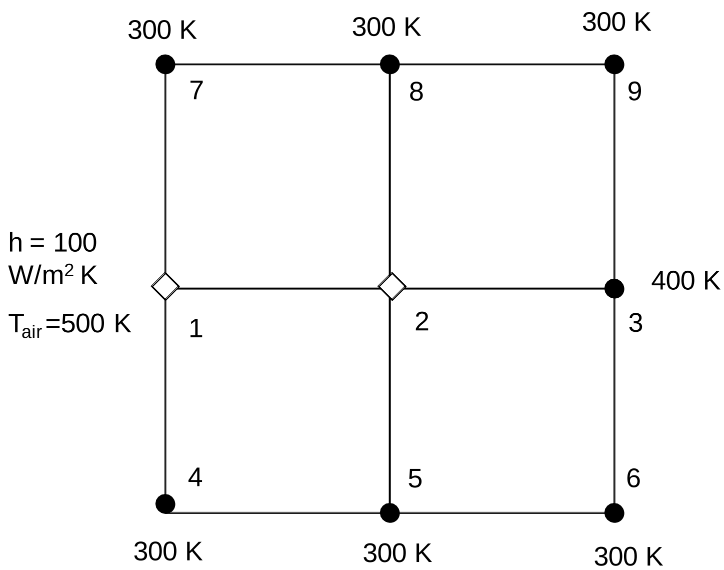
(iii) Determine the temperature at the center of the ball at $t = \tau_t$.

T = K

3. Consider unsteady heat conduction in a 2D square domain of side 2 m, as shown below. The domain is meshed with a square mesh with $\Delta x = \Delta y = 1$ m. The initial temperature at all grid points is 300 K. At time t=0, the boundary conditions shown below are applied. You are asked to determine how the temperatures of grid points 1 and 2 change with time using an explicit time stepping scheme.

You are given the following material properties:

$\rho = 2000$ kg/m^3 k=100 W/mK $C_p = 300$ J/kgK



(i) You are given the choice of two time steps, $\Delta t = 500$ s and $\Delta t = 2000$ s. Which one would you choose, and why?

$\Delta t =$ seconds Reason:

(ii) Develop analytical equations for T_1 and T_2 at time step $t = \Delta t$ in terms of neighbor temperatures, material properties and mesh parameters. Show analytical expressions for the discrete equations here.

$T_1 =$ $T_2 =$

(iii) Using the value of Δt you chose in part (i), determine the numerical values of the coefficients in the discrete equations derived in part (ii).

T_1 =
T_2 =

(iv) Find the numerical values of the temperatures T_1 and T_2 at $t = \Delta t$ using the explicit scheme.

$T_1 (t=\Delta t)$ =
$T_2 (t=\Delta t)$ =

BASIC EQUATION SHEET

Conservation Laws

Control Volume Energy Balance: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$; $\dot{E}_{st} = mC_p \, dT/dt$; $\dot{E}_{gen} = \dot{q}V$

Surface Energy Balance: $\dot{E}_{in} - \dot{E}_{out} = 0$

Conduction

Fourier's Law: $q''_{cond,x} = -k\frac{\partial T}{\partial x}$; $q''_{cond,n} = -k\frac{\partial T}{\partial n}$; $q_{cond} = q''_{cond}A$

Heat Flux Vector: $\overrightarrow{q''} = q''_x\vec{i} + q''_y\vec{j} + q''_z\vec{k} = -k\left[\frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}\right]$

Heat Diffusion Equation:

Rectangular Coordinates: $\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p\frac{\partial T}{\partial t}$

Cylindrical Coordinates: $\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p\frac{\partial T}{\partial t}$

Spherical Coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho C_p\frac{\partial T}{\partial t}$$

Thermal Resistance Concepts:

Conduction Resistance: $R_{t,cond} \overset{plane\,wall}{=} \frac{L}{kA}$; $R_{t,cond} \overset{cylinder}{=} \frac{\ln(r_o/r_i)}{2\pi lk}$; $R_{t,cond} \overset{sphere}{=} \frac{(1/r_i)-(1/r_o)}{4\pi k}$

Convection Resistance: $R_{t,conv} \overset{plane\,wall}{=} \frac{1}{h_{conv}A}$; $R_{t,conv} \overset{cylinder}{=} \frac{1}{2\pi rlh_{conv}}$; $R_{t,conv} \overset{sphere}{=} \frac{1}{4\pi r^2 h_{conv}}$

Radiation Resistance: $R_{t,rad} \overset{plane\,wall}{=} \frac{1}{h_{rad}A}$; $R_{t,rad} \overset{cylinder}{=} \frac{1}{2\pi rlh_{rad}}$; $R_{t,rad} \overset{sphere}{=} \frac{1}{4\pi r^2 h_{rad}}$

Combined Convection and Radiation Surface: $\frac{1}{R_{conv+rad}} = \frac{1}{R_{t,conv}} + \frac{1}{R_{t,rad}}$

Contact Resistance: $R_{t,contact} = \frac{1}{h_{contact}A_{contact}}$

Thermal Energy Generation:

$$T(x) - T_s \overset{plane\ wall}{=} \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right); \qquad T(r) - T_s \overset{cylinder}{=} \frac{\dot{q}r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right)$$

Extended Surfaces:

Convective Tip: $\dfrac{\theta(x)}{\theta_b}=\dfrac{\cosh\left[m(L-x)\right]+(h/mk)\sinh\left[m(L-x)\right]}{\cosh(mL)+(h/mk)\sinh(mL)}$

$$q_{fin}=\left(hPkA_c\right)^{1/2}\theta_b\frac{\sinh(mL)+(h/mk)\cosh(mL)}{\cosh(mL)+(h/mk)\sinh(mL)}$$

Adiabatic Tip: $\dfrac{\theta(x)}{\theta_b}=\dfrac{\cosh\left[m(L-x)\right]}{\cosh(mL)}$; $q_{fin}=\left(hPkA_c\right)^{1/2}\theta_b\tanh(mL)$

Prescribed Tip Temperature: $\dfrac{\theta(x)}{\theta_b}=\dfrac{(\theta_L/\theta_b)\sinh(mx)+\sinh\left[m(L-x)\right]}{\sinh(mL)}$

$$q_{fin}=\left(hPkA_c\right)^{1/2}\theta_b\frac{\cosh(mL)-(\theta_L/\theta_b)}{\sinh(mL)}$$

Infinitely Long Fin: $\dfrac{\theta(x)}{\theta_b}=e^{-mx}$; $q_{fin}=\left(hPkA_c\right)^{1/2}\theta_b$

$$m^2=\frac{hP}{kA_c};\ \theta_b=T_b-T_\infty\ ;\ q_{fin}=q_{conv,\,finsurface}+q_{conv,tip};\ q_{conv,tip}=hA_c\theta_L$$

Fin Effectiveness: $\varepsilon_{fin}=\dfrac{q_{fin}}{hA_{c,b}\theta_b}$; $\varepsilon_{fin}=\dfrac{R_{t,conv-base}}{R_{t,cond-fin}}$

Fin Efficiency: $\eta_{fin}=\dfrac{q_{fin}}{hA_{fin}\theta_b}$; $\eta_{fin}\overset{adiabatic}{=}\dfrac{\tanh(mL)}{mL}$; $L_c=L+\dfrac{A_c}{P}$; $\eta_{fin}=\dfrac{\tanh(mL_c)}{mL_c}$

$$\eta_o=\frac{q_{total}}{hA_{total}\theta_b}=1-\frac{NA_{fin}}{A_{total}}\left(1-\eta_{fin}\right);\ R_{t,cond-fin}=\frac{1}{\eta_{fin}hA_{fin}};\ R_{t,cond-fin\,array}=\frac{1}{\eta_o hA_{total}}$$

Two Dimensional Steady Conduction:

Conduction Shape Factor: $R_{t,cond}\overset{2D}{=}\dfrac{1}{Sk}$

Finite Difference Method: $T_{i+1,j}+T_{i-1,j}+T_{i,j+1}+T_{i,j-1}=4T_{i,j}$

Transient Conduction:

Lumped System Analysis: $Bi=\dfrac{R_{t-cond}}{R_{t-conv}}=\dfrac{h_{conv}L_c}{k_{solid}}$; $\dfrac{\theta}{\theta_i}=\dfrac{T-T_\infty}{T_i-T_\infty}=\exp\left(-\dfrac{t}{\tau_t}\right)$; $Fo=\dfrac{\alpha t}{L_c^2}$

$$\frac{\theta}{\theta_i}=\exp\left[-\left(\frac{h_{conv}L_c}{k_{solid}}\right)\left(\frac{\alpha t}{L_c^2}\right)\right]=\exp\left[-(Bi)(Fo)\right];\ \tau_t=\frac{\rho VC_p}{h_{conv}A_s}=C_{t,solid}R_{t,conv};$$

Analytical Solutions: $\theta^*=\dfrac{\theta}{\theta_i}=\dfrac{T-T_\infty}{T_i-T_\infty}$; $x^*=\dfrac{x}{L}$; $r^*=\dfrac{r}{r_o}$; $t^*=\dfrac{\alpha t}{L^2}$

Plane Wall: $\theta^*\underset{plane\,wall}{\cong}C_1\exp\left(-\zeta_1^2Fo\right)\cos\left(\zeta_1x^*\right)$; $\theta_o^*\overset{plane\,wall}{=}C_1\exp\left(-\zeta_1^2Fo\right)$; $\dfrac{Q}{Q_o}\overset{plane\,wall}{=}1-\theta_o^*\dfrac{\sin(\zeta_1)}{\zeta_1}$

Long Cylinder: $\theta^*\underset{cylinder}{\cong}C_1\exp\left(-\zeta_1^2Fo\right)J_0\left(\zeta_1r^*\right)$; $\theta_o^*\overset{cylinder}{=}C_1\exp\left(-\zeta_1^2Fo\right)$; $\dfrac{Q}{Q_o}\overset{cylinder}{=}1-2\theta_o^*\dfrac{J_1(\zeta_1)}{\zeta_1}$

Sphere: $\theta^* \underset{sphere}{\cong} C_1 \exp\left(-\zeta_1^2 Fo\right)\frac{\sin\left(\zeta_1 r^*\right)}{\zeta_1 r^*}$; $\theta_o^* \overset{sphere}{=} C_1 \exp\left(-\zeta_1^2 Fo\right)$;

$\frac{Q}{Q_o} \overset{sphere}{=} 1-3\theta_o^* \frac{\left[\sin\left(\zeta_1\right)-\zeta_1 \cos\left(\zeta_1\right)\right]}{\zeta_1^3}$

Table 5.1 from the textbook is attached at the end of the formula sheet.

Semi-infinite Solid:

Constant Surface Temperature: $\frac{T\left(x,t\right)-T_s}{T_i-T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$; $q_s'' = -k\left.\frac{\partial T}{\partial x}\right|_{x=0} = \frac{k\left(T_s-T_i\right)}{\sqrt{\pi\alpha t}}$

Constant Surface Heat Flux: $T\left(x,t\right)-T_i = \frac{2q_0''\left(\alpha t/\pi\right)^{1/2}}{k}\exp\left(-\frac{x^2}{4\alpha t}\right)-\frac{q_0''x}{k}erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$

Convection: $\frac{T\left(x,t\right)-T_i}{T_\infty-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)-\left[\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\right]\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$

Finite Difference Method:

For interior points on a uniform mesh:

Explicit Method: $T_{i,j}^{P+1} \overset{explicit}{=} \left(1-4Fo\right)T_{i,j}^P + Fo\left(T_{i+1,j}^P + T_{i-1,j}^P + T_{i,j+1}^P + T_{i,j-1}^P\right)$

Implicit Method: $T_{i,j}^{P} \overset{implicit}{=} \left(1+4Fo\right)T_{i,j}^{P+1} - Fo\left(T_{i+1,j}^{P+1} + T_{i-1,j}^{P+1} + T_{i,j+1}^{P+1} + T_{i,j-1}^{P+1}\right)$

Stability Limits: $\Delta t \underset{explicit}{\overset{1D}{\le}} \frac{\left(\Delta x\right)^2}{2\alpha}$; $\Delta t \underset{explicit}{\overset{2D}{\le}} \frac{\left(\Delta x\right)^2}{4\alpha}$; $\Delta t \underset{explicit}{\overset{3D}{\le}} \frac{\left(\Delta x\right)^2}{6\alpha}$

Convection

Newton's Law of Cooling: $q_{conv}'' = h_{conv}\left(T_s-T_\infty\right)$; $q_{conv} = q_{conv}''A$

Mass Transfer: $n_A'' = h_m\left(\rho_{A,s}-\rho_{A,\infty}\right)$; $q_{evap} = n_A''Ah_{fg}$; $\rho_A = M_A C_A$; $C_A = P_{A,sat}/R_u T$

Average Heat Transfer Coefficient: $\overline{h_{conv}} = \frac{1}{A_s}\int_{A_S} h_{conv}\,dA_s$

Average Mass Transfer Coefficient: $\overline{h_m} = \frac{1}{A_s}\int_{A_S} h_m\,dA_s$

Dimensionless Parameters:

Reynolds Number: $Re_{L_c} = \frac{\rho V L_c}{\mu} = \frac{V L_c}{\nu}$;

Prandtl Number: $Pr = \frac{\nu}{\alpha}$; Schmidt Number: $Sc = \frac{\nu}{D_{AB}}$; Lewis Number: $Le = \frac{\alpha}{D_{AB}}$

Nusselt Number: $Nu = \frac{h_{conv}L_c}{k_{fluid}}$; Sherwood Number: $Sh = \frac{h_m L_c}{D_{AB}}$

Boundary Layer Thickness: $\frac{\delta}{\delta_t} \approx Pr^n$; $\frac{\delta}{\delta_c} \approx Sc^n$; $\frac{\delta_t}{\delta_c} \approx Le^n$

Heat-Mass Analogy: $\frac{Nu}{Sh} = \frac{Pr^n}{Sc^n}$; $\frac{h}{h_m} = \frac{k}{D_{AB}Le^n} = \rho C_p Le^{1-n}$

Radiation

Emissive power = $E = \varepsilon\sigma T_s^4$

Irradiation received by surface from large surroundings: $G = \sigma T_{surr}^4$

Irradiation absorbed by surface = αG
Reflected irradiation: ρG

Gray surface: ε=α

Opaque surface: $\alpha + \rho = 1$

Radiative heat flux from a gray surface at T_S to a large surroundings at T_{surr}:

$$q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{surr}^4\right) = h_{rad}\left(T_s - T_{surr}\right)$$

$$h_{rad} = \varepsilon\sigma\left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right)$$

Useful Constants

σ = Stefan-Boltzmann's Constant = $5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\text{K}^4}$

R_u = Universal gas constant = 8314 J/kmolK

Geometry

Cylinder: $A = 2\pi rl$; $V = \pi r^2 l$

Sphere: $A = 4\pi r^2$; $V = \frac{4}{3}\pi r^3$

Triangle: *A= bh/2* *b:base h: height*

TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

	Plane Wall		Infinite Cylinder		Sphere	
Bi[a]	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2218	1.0082	0.3143	1.0124	0.3854	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3709	1.0173	0.4551	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6609	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8447	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	1.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2733	2.4050	1.6018	3.1415	2.0000

[a] $Bi = hL/k$ for the plane wall and hr_o/k for the infinite cylinder and sphere. See Figure 5.6.

Thermodynamic Properties of Saturated Water

Temperature, T (K)	Pressure, p (bars)[b]	Specific Volume (m³/kg) $v_f \cdot 10^3$	v_g	Heat of Vaporization, h_{fg} (kJ/kg)
273.15	0.00611	1.000	206.3	2502
275	0.00697	1.000	181.7	2497
280	0.00990	1.000	130.4	2485
285	0.01387	1.000	99.4	2473
290	0.01917	1.001	69.7	2461
295	0.02617	1.002	51.94	2449
300	0.03531	1.003	39.13	2438
305	0.04712	1.005	29.74	2426
310	0.06221	1.007	22.93	2414
315	0.08132	1.009	17.82	2402
320	0.1053	1.011	13.98	2390
325	0.1351	1.013	11.06	2378
330	0.1719	1.016	8.82	2366
335	0.2167	1.018	7.09	2354
340	0.2713	1.021	5.74	2342
345	0.3372	1.024	4.683	2329
350	0.4163	1.027	3.846	2317
355	0.5100	1.030	3.180	2304
360	0.6209	1.034	2.645	2291
365	0.7514	1.038	2.212	2278
370	0.9040	1.041	1.861	2265
373.15	1.0133	1.044	1.679	2257
375	1.0815	1.045	1.574	2252
380	1.2869	1.049	1.337	2239
385	1.5233	1.053	1.142	2225
390	1.794	1.058	0.980	2212
400	2.455	1.067	0.731	2183
410	3.302	1.077	0.553	2153