Write Down Your NAME

First Last

Circle Your DIVISION

Div. 1	**Div. 2**	**Div. 3**	**Div.4**
8:30 am	**9:30 pm**	**12:30 pm**	**3:30 pm**
Prof. Han	**Prof. Xu**	**Prof. Ruan**	**Prof. Pan**

ME315 Heat and Mass Transfer
School of Mechanical Engineering
Purdue University

Exam 2
November 16, 2017

Read Instructions Carefully:

- Write your name and circle your division number.
- Equation sheet and tables are attached to this exam. One page, letter-size, double-sided crib sheet is allowed.
- No books, notes, and other materials are allowed.
- ME Exam Calculator Policy is enforced. Only TI-30XIIS and TI-30XA are allowed.
- **Power off** all other digital devices, such as computer/tablet/phone and smart watch/glasses.
- Keep all the pages in order.
- You are asked to write your assumptions and answers to sub-problems in designated areas. Write on front side of the page only. If needed, you can insert extra pages but mark this clearly in the designated areas.

Performance		
1	**30**	
2	**35**	
3	**35**	
Total	**100**	

Problem 1

(1) An engineer conducts wind tunnel experiments on a model of an aircraft. The model has an identical geometry as the actual aircraft but its length is 1/5th of that of the actual aircraft. Assume that the air properties are the same in the wind tunnel and in the actual aircraft operation.

(a) If the actual aircraft experiences air flow velocity of U_2 while in operation, what should be the velocity of air (U_1) inside the wind tunnel to preserve aerodynamic similarity? [5 pts]

$$Re_1 = Re_2$$

$$\frac{\rho U_1 L_1}{\mu} = \frac{\rho U_2 L_2}{\mu} \Rightarrow \frac{\rho U_1 (\frac{1}{5} L_2)}{\mu} = \frac{\rho U_2 L_2}{\mu}$$

$$\Rightarrow U_1 = 5 U_2$$

(b) What will be the average heat transfer coefficient experienced by the actual aircraft if the measured heat transfer coefficient on the model inside the wind tunnel is $\bar{h}_1$? [5 pts]

$$\overline{Nu} = \frac{\bar{h} L}{k} = f(Re, Pr)$$

$$\because Re_1 = Re_2, \quad Pr_1 = Pr_2$$

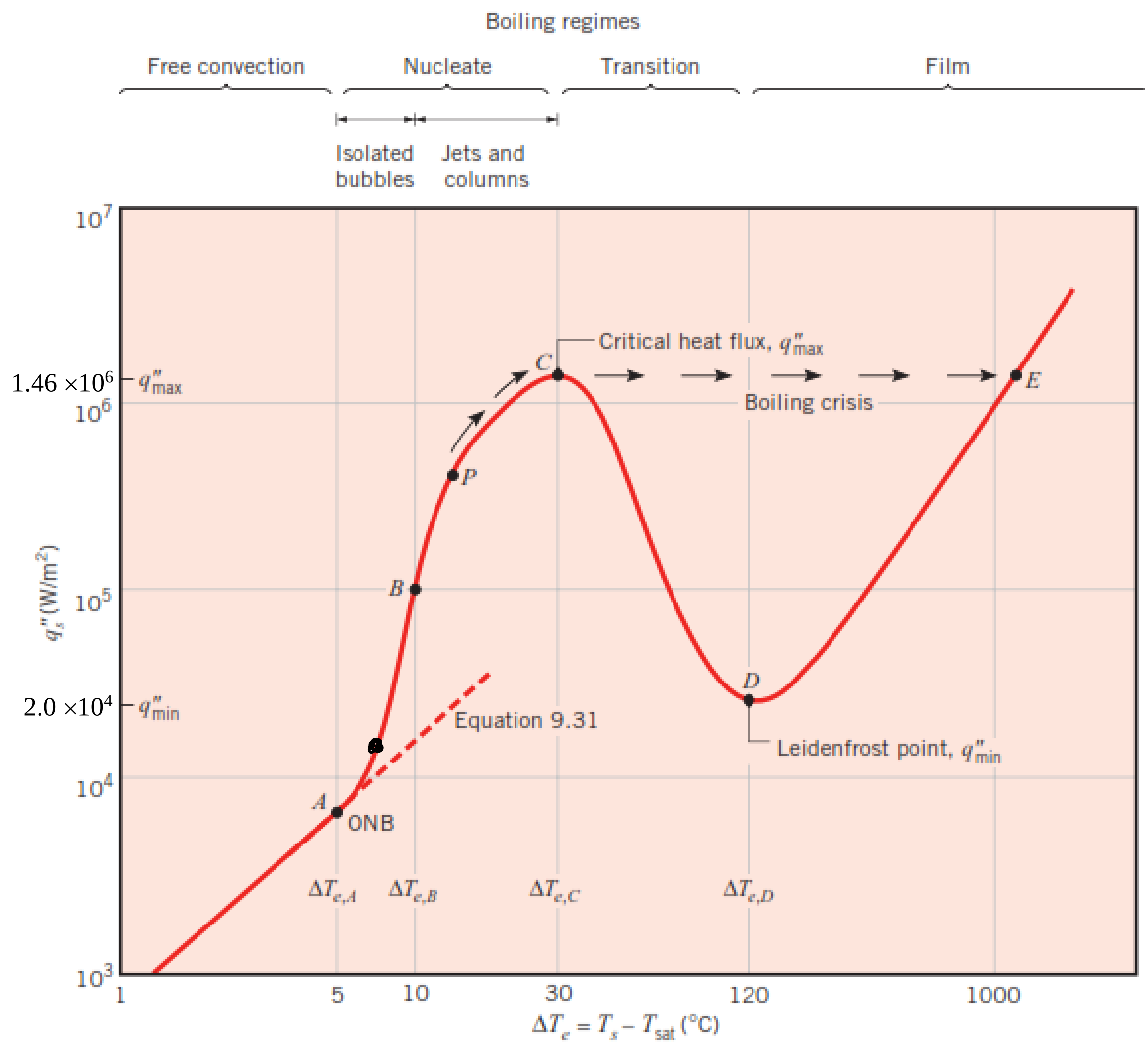
$$\therefore \overline{Nu}_1 = \overline{Nu}_2$$

$$\Rightarrow \frac{\bar{h}_1 L_1}{k} = \frac{\bar{h}_2 L_2}{k}$$

$$\Rightarrow \bar{h}_2 = \bar{h}_1 \cdot \frac{L_1}{L_2} = \frac{1}{5} \bar{h}_1$$

(2) Water boils in an electric hot pot with a heated area of 0.03 m^2 and with tunable power. Assume all the power consumed goes to boil the water. Let ΔT_e be the difference between the heated surface temperature and saturated water temperature.

(a) If the power is tuned to 500 W, use the boiling curve below to judge what boiling regime the hot pot is operated at. (5 pts)



$$q'' = \frac{500\ \text{W}}{0.03\ \text{m}^2} = 1.67 \times 10^4\ \text{W/m}^2$$

$$q'' < q''_{min}$$

It is in the nucleate boiling regime based on the boiling curve.

(b) If the power is gradually increased to 1,000W, will ΔT_e double as compared to the case in (a)? Briefly justify your answer. There is no need to calculate any values. (5 pts)

ΔT_e will not double.

because: $q_s'' = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{\frac{1}{2}} \left[\frac{c_{p,l}(T_s - T_{sat})}{C_{s,f} h_{fg} Pr_l^n}\right]^3 \sim \Delta T_e^3$

doubling q_s'' will not double ΔT_e.

or: From the boiling curve, doubling ΔT_e will lead to almost one order of magnitude increase in q_s''.

(3) Hot air enters a concentric counter flow heat exchanger at 200 °C and a mass flow rate of 0.1 kg/s, and heats 0.02 kg/s of engine oil entering the heat exchanger at 0 °C. The specific heat for air and water, $c_{p,air}$ = 1 kJ/kgK, and $c_{p,oil}$ = 2 kJ/kgK. What are the temperatures of air and oil exiting the heat exchanger if the heat exchanger is very very long? (10 pts)

$C_h = C_{p,h} \cdot \dot{m}_h = 0.1 \text{ kg/s} \times 1 \text{ kJ/kg}\cdot\text{K} = 0.1 \text{ kJ/s K}$

$C_c = C_{p,c} \cdot \dot{m}_c = 0.02 \text{ kg/s} \times 2 \text{ kJ/kg}\cdot\text{K} = 0.04 \text{ kJ/s}\cdot\text{K}$

Therefore $C_c = C_{min}$

for $L \to \infty$, $T_{c,o} = T_{h,i} = 200\,^\circ C$

From energy balance:

$C_h \cdot (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i})$

$0.1 (200 - T_{h,o}) = 0.04 (200 - 0)$

$T_{h,o} = 120\,^\circ C$

Problem 2 (35 points)

The main component of a solar water heater is a pipe whose surface absorbs solar irradiation. The pipe has a diameter of D and length L, and water enters the pipe with a mass flow rate $\dot{m}$ and inlet temperature $T_{m,i}$. On a sunny day the absorbed solar energy per unit length of the tube is q'. The following parameters are given. The length of the tube is longer than the hydrodynamic and thermal entry lengths.

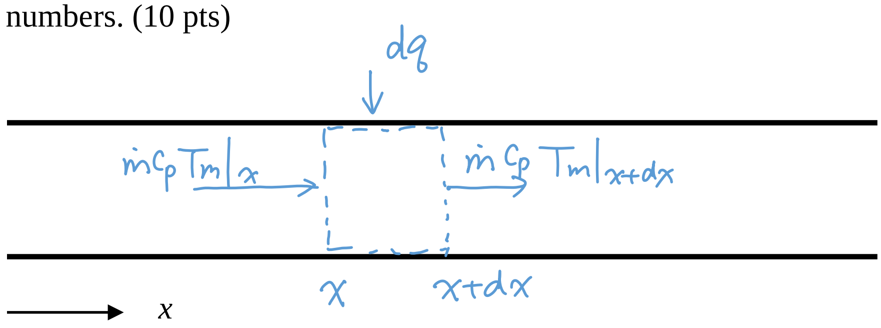
$D = 0.04$ m	$L = 10$ m	$\dot{m} = 0.002$ kg/s	$T_{m,i} = 20$ °C
$\rho = 1000$ kg/m^3	$\mu = 8.55\times10^{-4}$ N-s/m^2	$c_p = 4{,}200$ J/kg-K	$q' = 40$ W/m
$k = 0.613$ W/m-K			

TABLE 8.4 Summary of convection correlations for flow in a circular tube[a,b,e]

Correlation		Conditions
$f = 64/Re_D$	(8.19)	Laminar, fully developed
$Nu_D = 4.36$	(8.53)	Laminar, fully developed, uniform q_s''
$Nu_D = 3.66$	(8.55)	Laminar, fully developed, uniform T_s
$\overline{Nu}_D = 3.66 + \dfrac{0.0668\, Gz_D}{1 + 0.04\, Gz_D^{2/3}}$	(8.57)	Laminar, thermal entry (or combined entry with $Pr \gtrsim 5$), uniform T_s, $Gz_D = (D/x)\, Re_D\, Pr$
$\overline{Nu}_D = \dfrac{\dfrac{3.66}{\tanh[2.264\, Gz_D^{-1/3} + 1.7\, Gz_D^{-2/3}]} + 0.0499\, Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432\, Pr^{1/6}\, Gz_D^{-1/6})}$	(8.58)	Laminar, combined entry, $Pr \gtrsim 0.1$, uniform T_s, $Gz_D = (D/x)\, Re_D\, Pr$
$\dfrac{1}{\sqrt{f}} = -2.0 \log\left[\dfrac{e/D}{3.7} + \dfrac{2.51}{Re_D\sqrt{f}}\right]$	(8.20)[c]	Turbulent, fully developed
$f = (0.790 \ln Re_D - 1.64)^{-2}$	(8.21)[c]	Turbulent, fully developed, smooth walls, $3000 \lesssim Re_D \lesssim 5 \times 10^6$
$Nu_D = 0.023\, Re_D^{4/5}\, Pr^n$	(8.60)[d]	Turbulent, fully developed, $0.6 \lesssim Pr \lesssim 160$, $Re_D \gtrsim 10{,}000$, $(L/D) \gtrsim 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
$Nu_D = 0.027\, Re_D^{4/5}\, Pr^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.61)[d]	Turbulent, fully developed, $0.7 \lesssim Pr \lesssim 16{,}700$, $Re_D \gtrsim 10{,}000$, $L/D \gtrsim 10$
$Nu_D = \dfrac{(f/8)(Re_D - 1000)\, Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$	(8.62)[d]	Turbulent, fully developed, $0.5 \lesssim Pr \lesssim 2000$, $3000 \lesssim Re_D \lesssim 5 \times 10^6$, $(L/D) \gtrsim 10$
$Nu_D = 4.82 + 0.0185(Re_D\, Pr)^{0.827}$	(8.64)	Liquid metals, turbulent, fully developed, uniform q_s'', $3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5$, $3 \times 10^{-3} \lesssim Pr \lesssim 5 \times 10^{-2}$, $10^2 \lesssim Re_D\, Pr \lesssim 10^4$
$Nu_D = 5.0 + 0.025(Re_D\, Pr)^{0.8}$	(8.65)	Liquid metals, turbulent, fully developed, uniform T_s, $Re_D\, Pr \gtrsim 100$

[a]The mass transfer correlations may be obtained by replacing Nu_D and Pr by Sh_D and Sc, respectively.
[b]Properties in Equations 8.53, 8.55, 8.60, 8.61, 8.62, 8.64, and 8.65 are based on T_m; properties in Equations 8.19, 8.20, and 8.21 are based on $T_f = (T_s + T_m)/2$; properties in Equations 8.57 and 8.58 are based on $\overline{T}_m = (T_{m,i} + T_{m,o})/2$.
[c]Equation 8.20 pertains to smooth or rough tubes. Equation 8.21 pertains to smooth tubes.
[d]As a first approximation, Equations 8.60, 8.61, or 8.62 may be used to evaluate the average Nusselt number $\overline{Nu}_D$ over the entire tube length, if $(L/D) \gtrsim 10$. The properties should then be evaluated at the average of the mean temperature, $\overline{T}_m = (T_{m,i} + T_{m,o})/2$.
[e]For tubes of noncircular cross section, $Re_D \equiv D_h u_m/\nu$, $D_h \equiv 4A_c/P$, and $u_m = \dot{m}/\rho A_c$. Results for fully developed laminar flow are provided in Table 8.1. For turbulent flow, Equation 8.60 may be used as a first approximation.

(a) Using an appropriate differential control volume, derive a differential equation in terms of the given symbols that will allow you to solve the water mean temperature distribution $T_m(x)$ along the pipe. Neglect any radiation or convection loss to the ambient. Do not plug in any numbers. (10 pts)



$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

$$\dot{m} c_p T_m\big|_x + dq - \dot{m} c_p T_m\big|_{x+dx} = 0$$

$$dq = q' \, dx$$

$$T_m\big|_{x+dx} = T_m\big|_x + dT_m$$

$$\therefore \quad \dot{m} c_p T_m\big|_x + q' \, dx - \dot{m} c_p \left(T_m\big|_x + dT_m \right) = 0$$

$$\therefore \quad \dot{m} c_p \, dT_m = q' \, dx$$

(b) Calculate the water temperature at the outlet. (10 pts)

$$\dot{m} c_p \, dT_m = q' \, dx$$

$$dT_m = \frac{q'}{\dot{m} c_p} dx$$

$$\Rightarrow T_m(L) - T_{m,i} = \int_0^L \frac{q'}{\dot{m} c_p} dx$$

$$\Rightarrow T_m(L) = T_{m,i} + \frac{q' L}{\dot{m} c_p}$$

$$T_m(L) = 20^\circ C + \frac{40 W/m \cdot 10 m}{0.002 kg/s \times 4200 J/kg \cdot K}$$
$$= 67.6^\circ C$$

(c) Calculate the highest wall temperature along the tube. (15 pts)

The highest wall temperature will be at the outlet.

$$q'' = h(T_{s,o} - T_{m,o})$$

$$Re_D = \frac{4\dot{m}}{\pi \mu D} = \frac{4 \times 0.002 kg/s}{3.14 \times 8.55 \times 10^{-4} N \cdot s/m^2 \times 0.04 m} = 74.5$$

$\therefore$ Laminar, fully developed, uniform q_s''

$$Nu_D = 4.36 = \frac{hD}{k}$$

$$h = \frac{Nu_D k}{D} = \frac{4.36 \times 0.613 W/m \cdot K}{0.04 m} = 66.8 W/m^2 \cdot K$$

$$T_{s,o} = T_{m,o} + \frac{q''}{h} = T_{m,o} + \frac{q'/\pi D}{h}$$

$$= 67.6^\circ C + \frac{40 W/m}{3.14 \times 0.04 m \times 66.8 W/m^2 \cdot K} = 72.4^\circ C$$

Problem 3 (35 points)

It is known that for a spherical object with diameter D, the average Nusselt number can be expressed as: $\overline{Nu}_D = 2 + 0.6\, Re_D^{1/2} Pr^{1/3}$. Now consider a drop of water with diameter D that floats very slowly in air and thus its velocity can be considered as zero. The air is at temperature T_{air} and its relative humidity is ϕ.

(a) Express the average heat transfer coefficient $\overline{h}$ for the water droplet, in terms of given parameters and properties of water and/or air. **For each property used, list in the table below and state what it represents and at what temperature it is to be evaluated.** An example for viscosity μ is given in the table (this parameter may not may not be needed in your answer).

(b) Perform an energy balance analysis, and derive an expression for the steady-state temperature of the water droplet. **List each property you used, what it represents and at what temperature it is to be evaluated, in the same table below.**

Assumptions [3 pts] - List assumptions here

- heat-mass analogy applies
- steady state
- neglect radiation

Table of the Properties used in your answer, identify it is air or water properties or both, and at what temperature they are evaluated [5 pts]

μ	viscosity of air, evaluated at the film temperature
k_f	thermal conductivity of air at film temperature
D_{AB}	binary diffusion coefficient water/air, film temperature
$v_g(T_s)$	specific volumn of water vapor, at surface temperature
$v_g(T_{air})$	specific volumn of water vapor, at T_{air}
h_{fg}	latent heat of evaporation of water, at T_{surf}

Start your answer to question (a) here [10 pts]:

$\overline{Nu_D} = 2$ since $u = 0$

$\frac{\bar{h} \cdot D}{k_f} = 2$ $\bar{h} = \frac{2 k_f}{D}$

Start your answer to question (b) here [17 pts]:

From energy balance:

$\bar{h} (T_{air} - T_s) = \bar{h}_m (\rho_{A,s} - \rho_{A,\infty}) \cdot h_{fg}$

From Reynolds analogy,

$\overline{Sh_D} = 2$, $\frac{\bar{h}_m D}{D_{AB}} = 2$, $\bar{h}_m = \frac{2 D_{AB}}{D}$

$\frac{2 k_f}{D} (T_{air} - T_s) = \frac{2 D_{AB}}{D} \left(\frac{1}{v_g(T_s)} - \frac{\phi}{v_g(T_{air})} \right) \cdot h_{fg}$

Solve for T_s:

$T_s = T_{air} - \frac{D_{AB}}{k_f} \left(\frac{1}{v_g(T_s)} - \frac{\phi}{v_g(T_{air})} \right) \cdot h_{fg}$