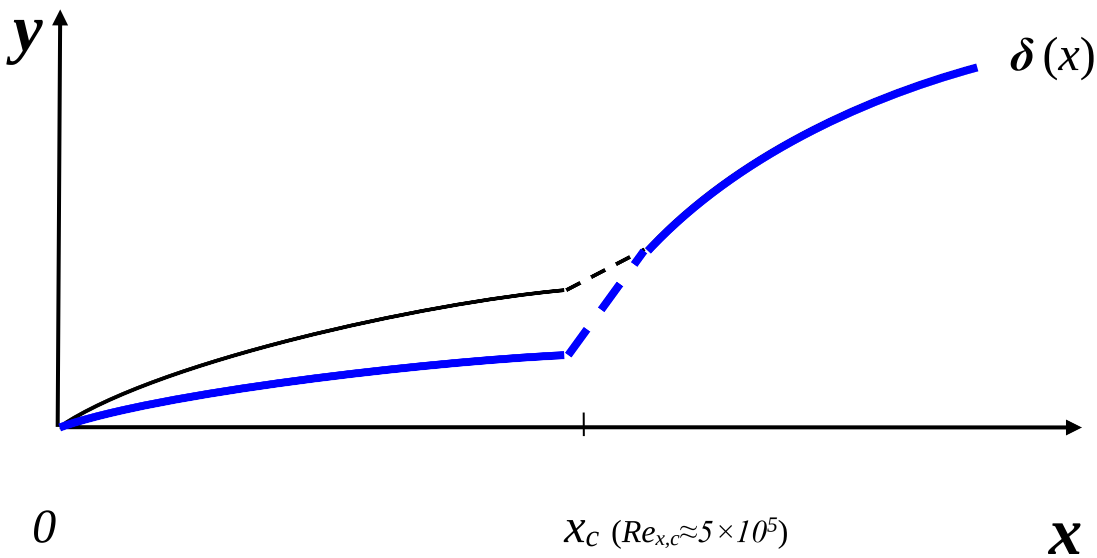
Circle your division: 1 2 3 4 Name________________________________

Problem 1 [35 points]

(a) (**9 pts**) Consider a cold fluid flowing over a flat plate of uniform surface temperature. The shape of velocity boundary layer (δ) is shown in the figure below. The Prandtl number (Pr) of the fluid is known to be 7. Recall that $\Pr = \nu/\alpha$. **<u>Qualitatively sketch the shape of thermal boundary layer</u> (δ_t)** relative to the velocity boundary layer (δ) in the figure below.



In the laminar region, the velocity and thermal boundary layer thicknesses have the relation of $\dfrac{\delta}{\delta_t} \approx Pr^n$, where $n \approx \dfrac{1}{3}$. For turbulence, the severe mixing leads to $\delta_t \approx \delta$.

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Problem 1 – continued

(b) (**8 pts**) Answer the following questions with <u>Ture (**T**) or False (**F**)</u>. There can be multiple correct answers. <u>And **briefly** justify your answers</u>.

(i) Which of the following is (are) true regarding entry *lengths for* ***laminar*** *fluid flow inside a circular tube?*

( ) The hydrodynamic entry length can be greater than the thermal entry length
( ) The hydrodynamic entry length can be equal to the thermal entry length
( ) The hydrodynamic entry length can be less than the thermal entry length

<u>**Briefly**</u> justify your answers in the box below:

The thermal boundary layer cannot be fully developed if the hydrodynamic boundary layer is not fully developed.

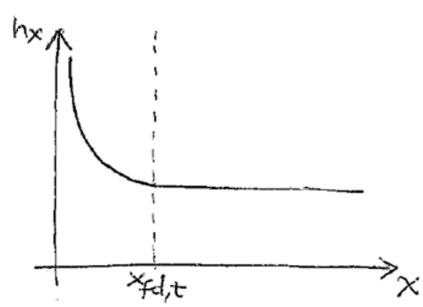
Answer: F, T, T

(ii) Which of the following is (are) true regarding Nusselt number for ***laminar*** *fluid flow inside a circular tube?*

( ) The average Nusselt number for the entire tube (including entry region) can be greater than the local Nusselt number for fully-developed conditions
( ) The average Nusselt number for the entire tube (including entry region) can be equal to the local Nusselt number for fully-developed conditions
( ) The average Nusselt number for the entire tube (including entry region) can be less than the local Nusselt number for fully-developed conditions

<u>**Briefly**</u> justify your answers in the box below:

The thermal entry region has a higher *h* than the thermal fully developed region.



Circle your division: 1 2 3 4 Name________________________________

Problem 1 – continued

(c) (**8 pts**) Suppose astronomers have found a planet that is likely inhabitable by humans. The temperature, pressure and composition of the atmosphere on that planet are the same as air on earth but the gravity is 50% less. You can treat people as cylinders and use the correlation $\overline{Nu} = \bar{h}L/k = C\,Ra_L^{1/4}$, where C is a constant and L is the height of the person.

Would people experience different heat transfer due to natural convection? Circle your answer below:

- *Higher* Heat Transfer Rate, i.e., $q_{\text{planet}} > q_{\text{earth}}$
- *Same* Heat Transfer Rate, i.e., $q_{\text{planet}} \approx q_{\text{earth}}$
- *Lower* Heat Transfer Rate, i.e., $q_{\text{planet}} < q_{\text{earth}}$

Briefly justify your answer in the box below.

From $Ra = \dfrac{g\,\beta\,\Delta T\,L_c^3}{\upsilon\alpha}$ where $\beta = \dfrac{1}{T}$

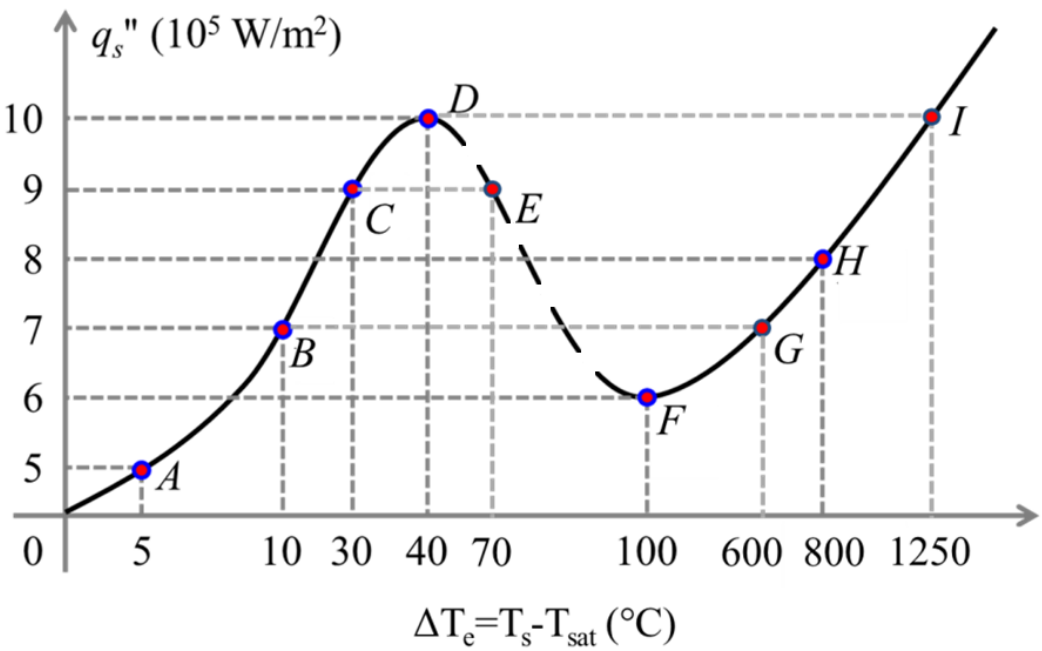
Due to less gravitational effect, the $Ra_{\text{planet}} < Ra_{\text{earth}}$

Therefore yields a smaller average Nu and lower overall heat transfer.

Circle your division: 1 2 3 4 Name________________________________

Problem 1 – continued

(d) (**10 pts**) A thin electrical chip is 1 cm × 1 cm in shape and has very small mass. It is cooled by saturated pool boiling of a dielectric fluid with a saturation temperature T_{sat} = 80 °C under atmospheric pressure. The boiling curve of the fluid is given below. If the chip normally generates 70 W of heat and the fluid is in nucleate boiling regime, the surface temperature of the chip is ____**90**___ **°C**. If the surface temperature of the chip cannot exceed 150 °C, the chip power cannot exceed ___**100**___ **W**.



Show your calculations in the box below:

$q = 70$ W

$q'' = q/A = 70/10^{-4} = 7\times10^5$ W/m²,

nucleate boiling ∴ point B.

$\Delta T_e = 10$ °C

$T_s = T_{sat} + \Delta T_e = 90$ °C

$q'' < q''_{CHF}$, $\Delta T_e < 40$ °C, $T_s < 120$ °C, $q'' > q''_{CHF}$, $\Delta T_e > 1250$ °C

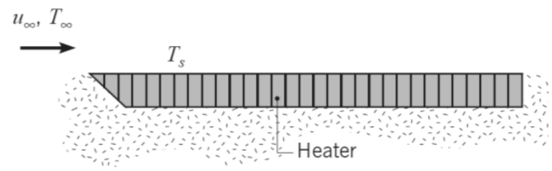
∴ $q''_{max} = q''_{CHF}$

$q''_{CHF} = 10\times10^5$ W/m² $q_{max} = q''_{CHF} \cdot A = 100$ W

Circle your division: 1 2 3 4 Name______________________________

Problem 2 [35 points]

Part I: **(20 pts)** A flat hot plate is exposed to dry, atmospheric air in parallel flow with $T_\infty = 300K$ and $u_\infty = 10m/s$. The plate is maintained at a uniform temperature of $T_S = 400K$ by an electrical heating element and is well insulated on the bottom. The plate has length of 0.5 m (along the flow direction) and width of 0.5 m (normal to the plane of the sketch). Thermophysical properties of dry air are listed as below. Assume steady state and neglect radiation.



Properties of Dry Air at Atmosphere Pressure

T(K)	v (m²/s)	k (W/mK)	Pr
300	15.89×10^{-6}	0.0263	0.707
350	20.92×10^{-6}	0.0300	0.700
400	26.41×10^{-6}	0.0338	0.690

(a) Is the flow laminar, turbulent or mixed? Justify your answer with calculations.

(b) Which location on the plate has the highest surface heat flux? Briefly justify your answer.

(c) Calculate the average Nusselt number ($\overline{Nu}_L$) for the plate.

(d) Calculate the total required heating power (W) to maintain steady state surface temperature.

Part II: **(15 pts)** The hot plate is now coated with a liquid layer of volatile species A under the same flow condition and T_S. Note that dry air has 0% relative humidity for species A. Thermophysical properties of spices A are listed as below.

Properties of Species A at Atmosphere Pressure

T(K)	h_{fg} (J/kg)	D_{AB} (m²/s)	Saturated vapor density $\rho_{sat,vapor}$ (kg/m³)
300	8×10^{6}	0.5×10^{-6}	0.08
350	9×10^{6}	1.0×10^{-6}	0.09
400	10×10^{6}	1.5×10^{-6}	0.10

(e) Identify a correlation for the average Sherwood number ($\overline{Sh}_L$) of this problem.

(f) Calculate the mass loss rate (kg/s) for species A.

(g) Calculate the total required heating power (W) after coating species A in order to maintain the same steady state surface temperature.

Circle your division: 1 2 3 4 Name________________________________

Problem 2 – continued

Part I:

Start your answer to part (a) here. [5 pts]

Assumptions: flat plate flow, incompressible, isothermal surface, etc…

Film temperature: $T_{film} = \frac{300+400}{2} = 350 \quad [K]$

$$\mathrm{Re}_L = \frac{u_\infty L}{\nu} = \frac{10\times 0.5}{20.92\times 10^{-6}} = 2.39\times 10^5 < 5\times 10^5$$

The flow is laminar

Start your answer to part (b) here. [5 pts]

The leading edge has higher heat loss. This can be explained through the correlations for local h_x,

where $h_x = Nu_x \frac{k_f}{x} = 0.332\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}\frac{k_f}{x} \propto \frac{\mathrm{Re}^{1/2}}{x} \propto x^{-1/2}$

Alternatively, we can explained through the fact that the thickness of boundary layer is thinner at the leading edge.

Start your answer to part (c) here. [5 pts]

$$\overline{Nu_L} = 0.664\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3} = 0.664\times(2.39\times 10^5)^{1/2}\times 0.7 = 288.2$$

Circle your division: 1 2 3 4 Name________________________________

Problem 2 – continued

Start your answer to part (d) here. [5 pts]

$$\bar{h} = \overline{Nu_L}\frac{k_f}{L} = 288.2 \times \frac{0.03}{0.5} = 17.3 \quad \left[\frac{W}{m^2K}\right]$$

$$q_{\text{cov}} = \bar{h}A(T_S - T_\infty) = 17.3 \times (0.5 \times 0.5) \times (400 - 300) = 432 \quad [W]$$

Part II:

Start your answer to part (e) here. [5 pts]

$$\overline{Sh_L} = 0.664\,\text{Re}_L^{1/2}\,Sc^{1/3}$$

$$= 0.664 \times (2.39 \times 10^5)^{1/2} \times \left(\frac{\nu}{D_{AB}}\right)^{1/3}$$

$$= 0.664 \times (2.39 \times 10^5)^{1/2} \times \left(\frac{20.92 \times 10^{-6}}{1 \times 10^{-6}}\right)^{1/3}$$

$$= 894$$

Start your answer to part (f) here. [5 pts]

$$\overline{h_m} = \overline{Sh_L}\frac{D_{AB}}{L} = 1.79 \times 10^{-3} \quad \left[\frac{m}{s}\right]$$

$$n_A = \overline{h_m}A\left(\rho_S - \rho_\infty\right) = 1.79 \times 10^{-3} \times 0.25 \times \left(0.1 - 0\right) = 4.47 \times 10^{-5} \quad \left[\frac{kg}{s}\right]$$

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Problem 2 – continued

Start your answer to part (g) here. [5 pts]

$$q_{evap} = n_A h_{fg} = 4.47 \times 10^{-5} \times 10 \times 10^6 = 447 \quad [W]$$

$$q_{total} = q_{evap} + q_{conv} = 879 \quad [W]$$

Circle your division: 1 2 3 4 Name________________________

Problem 3 [30 points]

Blood is warmed during open-heart surgery using a concentric tube, counter-flow heat exchanger in which water flows through the annular passage while blood flows through the inner tube. The heat exchanger length is L = 1 m and the thin-walled inner tube has diameter D = 50 mm. Water entering at 60°C with a mass flow rate of 0.1 kg/s is used for heating the blood entering at 28°C with a mass flow rate of 0.1 kg/s. The overall heat transfer coefficient (U) for the heat exchanger is 800 W/(m²·K). Assume specific heats for water and blood to be 4,200 J/(kg·K) and 3,500 J/(kg·K), respectively.

(a) Calculate the total rate of heat transfer (W) for the heat exchanger.

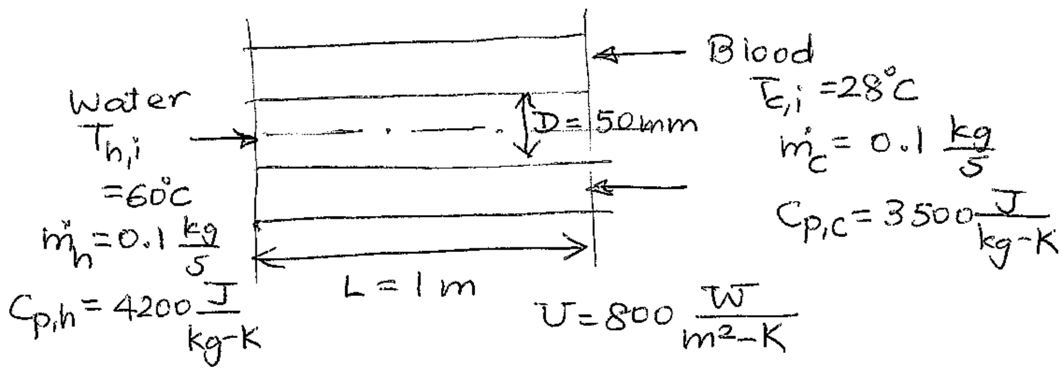
(b) Find the outlet temperatures (°C) of water and blood streams.

(c) Sketch the qualitative variation of the temperature of water and blood along the length of heat exchanger.

List your assumptions below. [2 pts]

* Steady state
* Uniform overall heat transfer coefficient
* Constant properties
* Ignore stray heat transfer

Start your answer to part (a) here. [15 pts]



Heat capacity rate of hot fluid (water)

$$C_h = \dot{m}_h C_{p,h} = 420 \frac{W}{K}$$

Heat capacity rate of cold fluid (blood)

$$C_c = \dot{m}_c C_{p,c} = 350 \frac{W}{K}$$

Heat capacity rate ratio : $C_r = \frac{C_{min}}{C_{max}} = \frac{350 \frac{W}{K}}{420 \frac{W}{K}} = 0.8333$

Number of transfer units : $NTU = \frac{UA}{C_{min}} = \frac{U(\pi D L)}{C_{min}}$

$$NTU = \frac{800 \frac{W}{m^2 K} * (\pi \times 1 \times 50 \times 10^{-3}) m^2}{350 \frac{W}{K}} = 0.359$$

Circle your division: 1 2 3 4 Name________________________________

Problem 3 – continued

Continue your answer to part (a) here.

For counter-flow heat exchanger: $\varepsilon = \dfrac{1 - \exp[-NTU(1-C_r)]}{1 - C_r \exp[-NTU(1-C_r)]}$

$$\Rightarrow \varepsilon = \frac{q}{q_{max}} \simeq 0.27$$

Rate of heat transfer: $q = 0.27\, q_{max}$

$$= 0.27\, C_{min}(T_{h,i} - T_{c,i})$$

$$= 0.27 \times 350 \frac{W}{K} \times (60-28)\,K$$

$$\boxed{q = 3024\ K}$$

Start your answer to part (b) here. [8 pts]

For the hot fluid: $q = q_h = C_h(T_{h,i} - T_{h,o})$

Outlet temperature of hot fluid (water): $T_{h,o} = T_{h,i} - \dfrac{q}{C_h}$

$$\boxed{T_{h,o} = 52.8^\circ C}$$

For the cold fluid: $q = q_c = C_c(T_{c,o} - T_{c,i})$

Outlet temperature of cold fluid (blood): $T_{c,o} = T_{c,i} + \dfrac{q}{C_c}$

$$\boxed{T_{c,o} = 36.64^\circ C}$$

Plot T vs. x in the figure below. [5 pts]