ME 315
Exam 2
Wednesday, November 11, 2015

- This is a closed-book, closed-notes examination. *There is a formula sheet provided. You are also allowed to bring your own one-page letter size, double-sided crib sheet.*
- You must turn off all communications devices before starting this exam, and leave them off for the entire exam.
- Please write legibly and show all work for your own benefit.
- State all assumptions.
- Please arrange all your sheets in the correct order.

Name: ____________________ ______________________
Last First

CIRCLE YOUR DIVISION

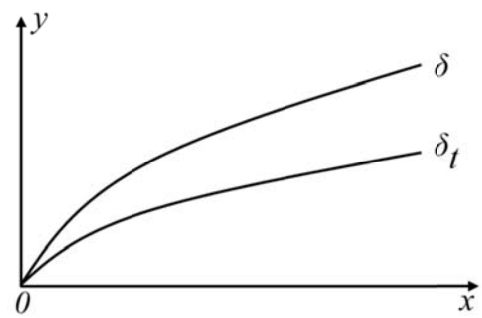
Div. 1 (8:30 am)	Div. 2 (9:30 am)	Div. 3 (11:30 am)	Div. 4 (3:30 pm)
Prof. Naik	Prof. Ruan	Prof. Pan	Prof. Marconnet

Your Assigned # : ________
(Only applicable to Div. 3)

Problem	Score
1 (30 Points)	
2 (40 Points)	
3 (30 Points)	
Total (100 Points)	

Problem 1 (30 pts)

(a) (**6 pts**) Consider a laminar flow over a flat plate. The shapes of thermal boundary layer (δ_t) and velocity boundary layers (δ) are given in the figure. The Prandtl number (Pr) should be ____.



<u>**A: Pr > 1**</u>
B: Pr ≈ 1
C: Pr < 1
D: Insufficient Information

<u>Briefly justify your choice.</u>

$$\frac{\delta}{\delta_t} = Pr^{n=1/3} > 1 \Rightarrow \underline{\boldsymbol{Pr > 1}}$$

<u>Alternatively</u>:
Prandtl number is the ratio of viscous diffusion rate compared to thermal diffusion rate; since viscous diffusion rate is higher, Prandtl number is greater than one

(b) (**7 pts**) The bottom surfaces of two identical cups of water (A and B) are heated to $T_{s,A}$ = 125°C and $T_{s,B}$ = 300°C. The boiling curve for saturated water at atmospheric pressure is given below. Note: the excess temperature ΔT_e is defined as $T_s - T_{sat}$, where T_{sat} for water at atmospheric pressure is 100°C.

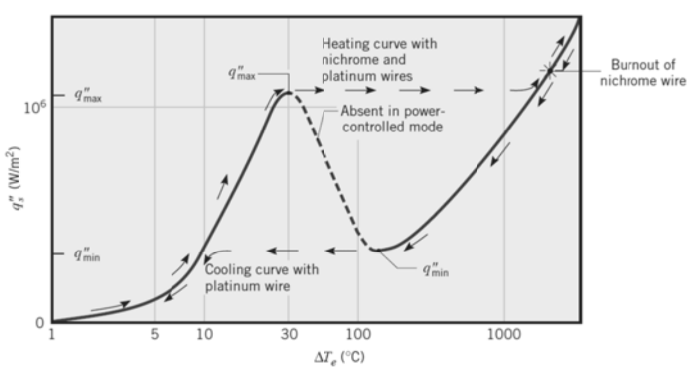
Which cup of water has a higher rate of evaporation? <u>Qualitatively justify your answer using the boiling curve, your understanding of the boiling regimes, and/or appropriate equations. No calculations are required.</u>

From the pool boiling curve:

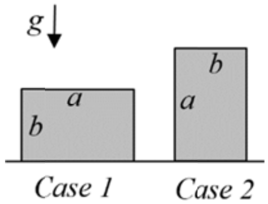
$\Delta T_{e,A} = 25^\circ \mathrm{C} \Rightarrow$ nucleate boiling region near critical heat flux providing very high rate of heat transfer

$\Delta T_{e,B} = 200^\circ \mathrm{C} \Rightarrow$ film boiling region near minimum heat flux (Leidenfrost point) providing significantly less rate of heat transfer

$\Rightarrow$ <u>**Cup A has higher rate of evaporation**</u>



(c) (**7 pts**) A hot rectangular plate, of length ***a*** and width ***b*** with ***a*** > ***b***, is hung vertically in air and cooled by free convection. The vertical side can be either along ***b*** (Case 1) or ***a*** (Case 2), as illustrated. Assume the boundary layer for free convection is laminar. Recall that the correlation of average Nusselt number in laminar region has the form:



$$\overline{Nu} = \frac{\bar{h}\,L}{k} = CRa_L^{1/4}$$, where C is a constant.

Briefly explain whether the orientation of the plate will affect the cooling rate.

$\overline{Nu} \propto Ra_L^{1/4} \propto Gr_L^{1/4} \propto L^{3/4} \Rightarrow \bar{h} \propto \frac{L^{3/4}}{L} \propto L^{-1/4} \Rightarrow$ average heat transfer coefficient decreases with increase in length of the plate $\Rightarrow$ with **shorter side (*b*) along the vertical will provide faster cooling rate**

Alternatively:

Boundary layer grows along the vertical side. With the longer side (a) along the vertical, a thicker boundary layer will result in lower value of heat transfer coefficient $\Rightarrow$ with shorter side (b) along the vertical will provide faster cooling rate

(d) (**10 pts**) A concentric-tube heat exchanger is designed to work with a hot liquid and a cold liquid with known inlet temperatures. It can be operated in either parallel-flow or counter-flow configuration. The heat exchanger has the following parameters:

Overall heat transfer coefficient: U = 100 W/(m^{2}K); Area: A = 1 m^2

Heat Capacity Rates: $C_h = C_c = 100$ W/K

Calculate the effectiveness (ε) of parallel flow and counter-flow configurations and identify which flow configuration will be more effective.

$$NTU = \frac{UA}{C_{min}} = \frac{100\,\frac{\mathrm{W}}{\mathrm{m^2\text{-}K}} \times 1\ \mathrm{m}^2}{100\,\frac{\mathrm{W}}{\mathrm{K}}} = 1 \ \text{ and } \ C_r = \frac{C_{min}}{C_{max}} = \frac{100\,\frac{\mathrm{W}}{\mathrm{K}}}{100\,\frac{\mathrm{W}}{\mathrm{K}}} = 1$$

For parallel flow: $\varepsilon_{HEX,PF} = \dfrac{1-\exp\left[-NTU\left(1+C_r\right)\right]}{\left(1+C_r\right)} = 0.432$

Alternatively:

Figure 11.10 $\varepsilon_{HEX,PF} \cong 0.42$

For counter-flow: $\varepsilon_{HEX,CF} = \dfrac{NTU}{\left(1+NTU\right)} = 0.5$

Alternatively:

Figure 11.11 $\varepsilon_{HEX,CF} \cong 0.5$

$\Rightarrow$ **counter-flow arrangement is more effective**

Problem 2 (40 pts)

Consider a chilly, autumn day at Purdue with wind velocity of U_∞ = 2 m/s and an ambient temperature of T_∞ = 10°C. You forget your jacket at home and your forearms are exposed to the cold air. For the air, assume k = 0.026 W/(m-K), Pr = 0.7, and the viscosity given by the table. Recall that $\alpha = k / \rho c_p = \nu / Pr$.

	ν [m^2/s]
Air @ 10°C	1.35×10^{-5}
Air @ 23.5°C	1.52×10^{-5}
Air @ 37°C	1.68×10^{-5}

For this problem, approximate your forearm as a long cylinder, with diameter D = 75 mm, in cross-flow of air with a surface temperature of T_s = 37°C.

(a) (**13 pts**) Calculate the rate of heat loss per unit area from your arm.

Film temperature: $T_{film} = \dfrac{T_s + T_\infty}{2} = 23.5^\circ\text{C}$

Reynolds number: $Re_D = \dfrac{u_\infty D}{\nu_{air}} = \dfrac{2\,\frac{\text{m}}{\text{s}} \times 75 \times 10^{-3}\,\text{m}}{1.52 \times 10^{-5}\,\frac{\text{m}^2}{\text{s}}} = 9868.4$

For cylinder in cross-flow with air: $\overline{Nu_D} = CRe_D^m Pr^{1/3} = 0.193 Re_D^{0.618} Pr^{1/3} = 50.4$

$$\overline{Nu_D} = \frac{\bar{h}D}{k_{air}} \Rightarrow 50.4 = \frac{\bar{h} \times 75 \times 10^{-3}\,\text{m}}{0.026\,\frac{\text{W}}{\text{m-K}}} \Rightarrow \bar{h} = 17.5\,\frac{\text{W}}{\text{m}^2\text{-K}}$$

Rate of heat loss per unit area: $q''_{conv} = \bar{h}(T_s - T_\infty) = 17.5\,\dfrac{\text{W}}{\text{m}^2\text{-K}} \times (37 - 10)\,\text{K} \Rightarrow \underline{\boldsymbol{q''_{conv} = 472.5\,\frac{\textbf{W}}{\textbf{m}^2}}}$

(b) (**20 pts**) Now you run through the fountain outside of ME and your arms are coated uniformly with a thin layer of water (D_{AB} = 2.17 × 10^{-5} m^2/s, h_{fg} = 2.4 × 10^{6}J/kg). Assume the surrounding air has a relativity humidity of ϕ_∞ = *80%*. Calculate the total rate of heat per unit area for your wet forearms.

	$\rho_{sat,vapor}$ [kg/m^3]
Water @ 10°C	1.0×10^{-2}
Water @ 23.5°C	1.9×10^{-2}
Water @ 37°C	4.4×10^{-2}

Schmidt number: $Sc = \dfrac{\nu_{air}}{D_{AB}} = \dfrac{1.52 \times 10^{-5}\,\frac{\text{m}^2}{\text{s}}}{2.17 \times 10^{-5}\,\frac{\text{m}^2}{\text{s}}} = 0.7$

Assume heat and mass transfer analogy is applicable

$$\Rightarrow \overline{Sh_D} = CRe_D^m Sc^{1/3} = 0.193 Re_D^{0.618} Sc^{1/3} = 50.4$$

$$\overline{Sh_D} = \frac{\overline{h_m} D}{D_{AB}} \Rightarrow 50.4 = \frac{\overline{h_m} \times 75\times10^{-3}\,\text{m}}{2.17\times10^{-5}\,\frac{\text{m}^2}{\text{s}}} \Rightarrow \overline{h_m} = 0.0146\,\frac{\text{m}}{\text{s}}$$

<u>Alternatively</u>:

$$Le = \frac{\alpha}{D_{AB}} = \frac{\nu}{Pr\times D_{AB}} = \frac{1.52\times10^{-5}\,\frac{\text{m}^2}{\text{s}}}{0.7\times2.17\times10^{-5}\,\frac{\text{m}^2}{\text{s}}} = 1\,;\ \frac{\overline{h}}{\overline{h_m}} = \frac{k}{D_{AB} Le^{n=1/3}}$$

$$\Rightarrow \frac{17.5\,\frac{\text{W}}{\text{m}^2\text{-K}}}{\overline{h_m}} = \frac{0.026\,\frac{\text{W}}{\text{m-K}}}{2.17\times10^{-5}\,\frac{\text{m}^2}{\text{s}}\times1^{1/3}} \Rightarrow \overline{h_m} = 0.0146\,\frac{\text{m}}{\text{s}}$$

Rate of evaporation of water vapor from the surface:

$$\dot{m}_{evap} = \overline{h_m} A\left(\rho_{A,s} - \rho_{A,\infty}\right) = \overline{h_m} A\left(\rho_{A,sat}\left(T_s\right) - \phi_\infty \rho_{A,sat}\left(T_\infty\right)\right) = 0.0146\,\frac{\text{m}}{\text{s}} \times \left(4.4\times10^{-2} - 0.8\times10^{-2}\right)\frac{\text{kg}}{\text{m}^3}$$

$$\dot{m}_{evap} = 5.26\times10^{-4}\,\frac{\text{kg}}{\text{m}^2\text{-s}}$$

Rate of evaporative heat loss per unit area:

$$q''_{evap} = \dot{m}_{evap} h_{fg} = 5.26\times10^{-4}\,\frac{\text{kg}}{\text{m}^2\text{-s}} \times 2.4\times10^6\,\frac{\text{J}}{\text{kg}} = 1262.4\,\frac{\text{W}}{\text{m}^2}$$

Total rate of heat loss per unit area: $q''_{total} = q''_{conv} + q''_{evap} \Rightarrow \underline{\boldsymbol{q''_{total} = 1734.9\,\frac{W}{m^2}}}$

(c) (**7 pts**) Now consider that you were wearing a jacket of low thermal conductivity material that fits snugly over your forearm adding an effective thermal resistance of *R''* = 0.05 m^2-K/W. Assume your arm and jacket are dry and that the thickness of your jacket does not change the convective heat transfer coefficient significantly from part (a). Find the rate of heat loss per unit area from your arm with the jacket.

Thermal resistance due to convection: $R''_{conv} = \frac{1}{\overline{h}} = 0.057\,\frac{\text{m}^2\text{-K}}{\text{W}}$

Total thermal resistance due to convection and conduction through the jacket:

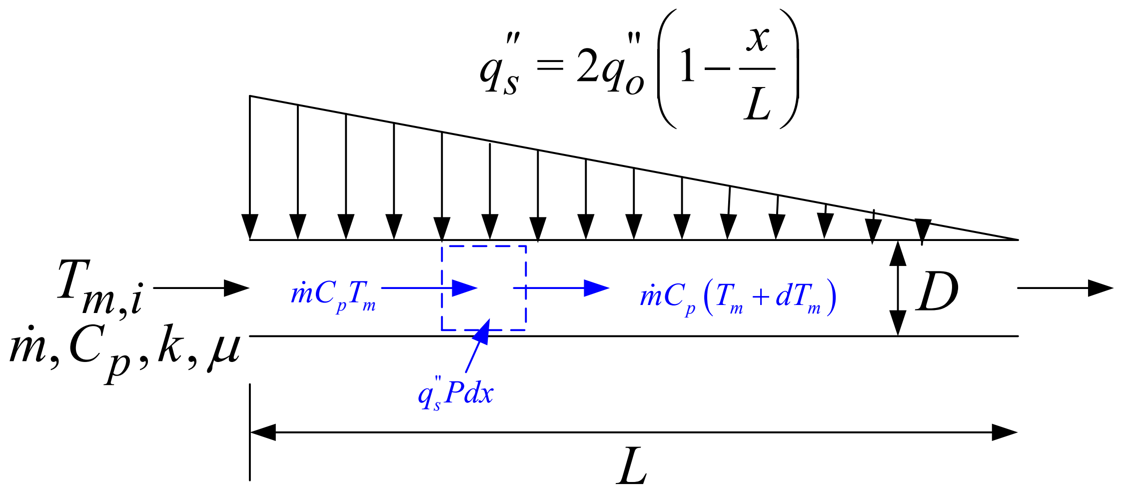
$$R''_{total} = R'' + R''_{conv} = \left(0.05 + 0.57\right)\frac{\text{m}^2\text{-K}}{\text{W}} = 0.107\,\frac{\text{m}^2\text{-K}}{\text{W}}$$

Rate of heat loss per unit area with the jacket: $q''_{conv,new} = \frac{\left(T_s - T_\infty\right)}{R''_{total}} = \frac{\left(37-10\right)\text{K}}{0.107\,\frac{\text{m}^2\text{-K}}{\text{W}}} \Rightarrow$

$$\underline{\boldsymbol{q''_{conv,new} = 252.3\,\frac{W}{m^2}}}$$

Problem 3 (30 pts)

A fluid of specific heat C_p, thermal conductivity k, and viscosity μ flows steadily through a circular tube of diameter D and length L. The mass flow rate is $\dot{m}$ and the fluid enters with a mean temperature of $T_{m,i}$. The flow is turbulent and fully-developed over the entire length of the tube. The tube surface (wall) is subjected to a heat flux that decreases linearly from inlet to outlet as: $q_s'' = 2q_o''\left(1-\frac{x}{L}\right)$ where $q_o'' > 0$ is known.



(a) (**4 pts**) Using an appropriate correlation, write an expression for convective heat transfer coefficient h inside the tube only in terms of known parameters.

For turbulent, fully-developed flow with fluid heating: $\overline{Nu_D} = \frac{\bar{h}D}{k} = 0.023Re_D^{0.8}Pr^{0.4}$

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{\rho D}{\mu}\left(\frac{\dot{m}}{\frac{\pi}{4}D^2\rho}\right) = \frac{4\dot{m}}{\pi D\mu} \text{ and } Pr = \frac{\nu}{\alpha} = \frac{\mu}{\rho}\frac{\rho C_p}{k} = \frac{\mu C_p}{k}$$

(b) (**10 pts**) Derive an expression for variation of the mean fluid temperature $T_m(x)$ only in terms of known parameters.

Considering energy balance for the control volume: $\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_{gen}} = \cancel{\dot{E}_{st}}$

$$\Rightarrow \dot{m}C_p T_m + q_s'' P dx - \dot{m}C_p\left(T_m + dT_m\right) = 0 \Rightarrow \dot{m}C_p dT_m = q_s'' P dx$$

Integrating: $\int_{T_{m,i}}^{T_m(x)} dT_m = \frac{2q_o'' P}{\dot{m}C_p}\int_0^x \left(1 - \frac{x}{L}\right)dx \Rightarrow \underline{\boldsymbol{T_m(x) = T_{m,i} + \frac{2q_o'' P}{\dot{m}C_p}\left(x - \frac{x^2}{2L}\right)}}\boldsymbol{;\ P = \pi D}$

(c) (**6 pts**) Derive an expression for the total rate of heat transfer q to the fluid only in terms of known parameters.

Total rate of heat transfer: $q = \dot{m}C_p\left(T_{m,o} - T_{m,i}\right)$

$$T_{m,o} = T_{m,i} + \frac{2q_o'' P}{\dot{m}C_p}\left(L - \frac{L^2}{2L}\right) = T_{m,i} + \frac{q_o'' PL}{\dot{m}C_p} \Rightarrow q = \dot{m}C_p\left(T_{m,i} + \frac{q_o'' PL}{\dot{m}C_p} - T_{m,i}\right) = q_o'' PL$$

$\boldsymbol{q = q_o'' \pi DL}$

<u>Alternatively</u>:

$$q = \int_0^L q_s'' P dx = \int_0^L 2q_o''\left(1 - \frac{x}{L}\right)(\pi D)dx = 2q_o'' \pi D\left(x - \frac{x^2}{2L}\right)_0^L = 2q_o'' \pi D\left(L - \frac{L^2}{2L}\right) = q_o'' \pi DL$$

(d) (**5 pts**) Derive an expression for variation of the surface (wall) temperature $T_s(x)$ only in terms of known parameters.

Considering heat flux at any section: $q_s'' = \bar{h}\left[T_s(x) - T_m(x)\right] \Rightarrow T_s(x) = T_m(x) + \dfrac{q_s''}{\bar{h}}$

$$\boldsymbol{T_s(x) = T_{m,i} + \frac{2q_o''P}{\dot{m}C_p}\left(x - \frac{x^2}{2L}\right) + \frac{2q_o''}{\bar{h}}\left(1 - \frac{x}{L}\right)}$$

(e) (**5 pts**) Derive an expression for the axial location x_{max} at which the surface (wall) temperature is maximum.

For maximum surface temperature: $\dfrac{dT_s}{dx} = 0 = \dfrac{2q_o''P}{\dot{m}C_p} - \dfrac{2q_o''P}{\dot{m}C_p}\dfrac{x_{max}}{L} - \dfrac{2q_o''}{\bar{h}L}$

$$\Rightarrow \frac{P}{\dot{m}C_p}\frac{x_{max}}{L} = \frac{P}{\dot{m}C_p} - \frac{1}{\bar{h}L} \Rightarrow \underline{\boldsymbol{x_{max} = L - \frac{\dot{m}C_p}{\pi D\bar{h}}}} \text{ for } \underline{\boldsymbol{L > \frac{\dot{m}C_p}{\pi D\bar{h}}}}$$