**Name:**___________________________________
**Last** **First**

**CIRCLE YOUR DIVISION:**

| | | |
|---|---|---|
| **Div. 1 (9:30 am)** | **Div. 2 (11:30 am)** | **Div. 3 (2:30 pm)** |
| **Prof. Ruan** | **Prof. Naik** | **Mr. Singh** |

**School of Mechanical Engineering**
**Purdue University**
**ME315 Heat and Mass Transfer**

# Exam #2

**Wednesday, October 20, 2010**

**Instructions:**

- Write your name on each page
- Closed-book exam – a list of equations is given
- Please write legibly and show all work for your own benefit. Write on one side of the page only.
- Keep all pages in order
- You are asked to write your assumptions and answers to sub-problems in designated areas. Only the work in its designated area will be graded.

**Performance**

| | | |
|---|---|---|
| **1** | **35** | |
| **2** | **30** | |
| **3** | **35** | |
| **Total** | **100** | |

**Name:____________________________________**

**Last** **First**

**Problem 1 [35 pts]**

A sphere of diameter 18 mm has thermal conductivity of 10 W/m-K, density of 7800 kg/m$^3$, and specific heat of 400 J/kg-K. Initially the sphere is at a uniform temperature of 27°C. The sphere is exposed to air at $T_\infty$ = 2000°C with convective heat transfer coefficient $h_{conv}$ = 100 W/m$^2$-K. The emissivity of the sphere surface (considered to be gray) is 0.8. The walls of the furnace are at $T_{surr}$ = 2000°C and can be considered large relative to the size of the sphere.

(a) Neglecting radiation effects, calculate the time required (seconds) for the temperature of the sphere to reach 500°C.

(b) Now consider that radiation from the furnace walls cannot be neglected. How much time (seconds) is required to reach 500°C at the center of the sphere? Comment on the time to reach 500°C in parts (a) and (b).

In part (b), assume that radiation can be treated linearly and that the radiative heat transfer coefficient can be calculated based on initial temperature of the sphere, neglecting the variation of radiative heat transfer coefficient with changing surface temperature.

**List your assumptions here [3 pts]:**

One-dimensional radial conduction in the sphere

Constant properties

Uniform radiative heat transfer coefficient (remains constant) on the surface

**Start your answer to part (a) here [12 pts]:**

Biot number: $Bi = \dfrac{h_{conv}L_c}{k_{solid}} = \dfrac{100\,\dfrac{\text{W}}{\text{m}^2\text{-K}} \times \left(\dfrac{3}{1000}\right)\text{m}}{10\,\dfrac{\text{W}}{\text{m-K}}} = 0.03;\ L_c = \dfrac{V}{A_s} = \dfrac{D}{6}$

$\Rightarrow$ lumped capacitance approach is valid ($Bi < 0.1$)

Temperature distribution through the sphere: $\dfrac{T - T_\infty}{T_i - T_\infty} = \exp(-Bi \times Fo)$

$\Rightarrow \dfrac{500 - 2000}{27 - 2000} = \exp(-0.03 \times Fo)$ Solving we get: $Fo = 9.14 = \dfrac{\alpha t}{L_c^2}$

Thermal diffusivity: $\alpha = \dfrac{k}{\rho C_p} = 3.205 \times 10^{-6}\,\dfrac{\text{m}^2}{\text{s}}$

Therefore, the time required to reach 500°C: $\underline{\boldsymbol{t = 25.7}\ \textbf{seconds}}$

Name:______________________________

Last First

**Problem 1 - cont.**

**Start your answer to part (b) here [20 pts]:**

Radiative heat transfer coefficient: $h_{rad} = \varepsilon\sigma\left(T_s + T_{surr}\right)\left(T_s^2 + T_{surr}^2\right)$

$$h_{rad} = 0.8\times5.67\times10^{-8}\,\frac{\text{W}}{\text{m}^2\text{-K}^4}\times\left(300+2273\right)\text{K}\times\left(\left(300\right)^2+\left(2273\right)^2\right)\text{K}^2 = 613.5\,\frac{\text{W}}{\text{m}^2\text{-K}}$$

Convection and radiative effects are additive at the surface: $h_{total} = 713.5\,\dfrac{\text{W}}{\text{m}^2\text{-K}}$

Biot number: $Bi = \dfrac{h_{total}L_c}{k_{solid}} = \dfrac{713.5\,\dfrac{\text{W}}{\text{m}^2\text{-K}}\times\left(\dfrac{3}{1000}\right)\text{m}}{10\,\dfrac{\text{W}}{\text{m-K}}} = 0.21405;\ L_c = \dfrac{V}{A_s} = \dfrac{D}{6}$

$\Rightarrow$ lumped capacitance approach is not valid ($Bi > 0.1$)

Non-dimensional temperature at the center of the sphere: $\theta_o^* = \dfrac{T_o - T_\infty}{T_i - T_\infty} = C_1\exp\left(-\xi_1^2 Fo\right)$

Using Table 5.1: $C_1 = 1.1825$ and $\xi_1 = 1.3015$ for $Bi = \dfrac{h_{total}r_o}{k_{solid}} = \dfrac{713.5\,\dfrac{\text{W}}{\text{m}^2\text{-K}}\times\left(\dfrac{9}{1000}\right)\text{m}}{10\,\dfrac{\text{W}}{\text{m-K}}} = 0.64215$

$\Rightarrow \dfrac{500-2000}{27-2000} = 1.825\exp\left(-\left(1.3015\right)^2\times Fo\right)$ Solving we get: $Fo = 0.2608 = \dfrac{\alpha t}{r_o^2}$

Therefore, the time required to reach 500°C at the center of the sphere: ***t* = 6.6 seconds**

**Since total rate of heat transfer to the sphere increases when considering radiation from the furnace walls in addition to convection from the ambient air, less time is required to attain the same temperature of 500°C at the center of the sphere.**

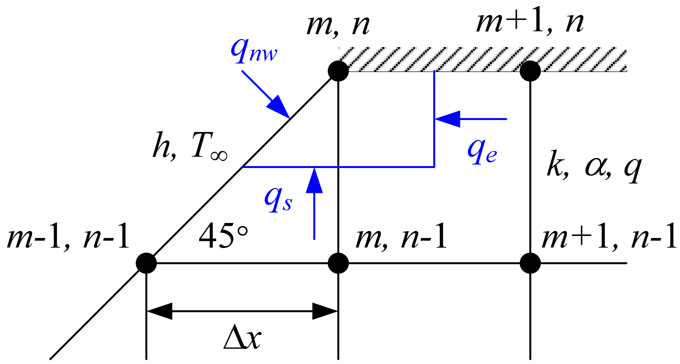
Name:______________________________

Last First

**Problem 2 [30 pts]**

Consider a two-dimensional uniform mesh with cell width $\Delta x = \Delta y$ shown below. The solid has thermal conductivity $k$, thermal diffusivity $\alpha$, and generates heat at a uniform volumetric rate of $\dot{q}$. You need to develop the transient nodal equation for a corner node ($m$, $n$) using energy balance method. The top boundary is perfectly insulated, while the inclined boundary is exposed to a fluid at $T_\infty$ with a convective heat transfer coefficient $h$.

(a) Using the explicit method derive an expression for $T_{m,n}^{p+1}$ in terms of $T_{m,n}^{p}$ and the temperatures of the relevant surrounding nodes as well as the temperature of ambient fluid. Express your answer in terms of the finite difference Biot number ($Bi$) the finite difference Fourier number ($Fo$) as well as any other given parameters.




(b) Derive the stability criterion for the nodal equation developed in (a).

**Start you answer to part (a) here [20 pts]:**

Considering energy balance for the control volume shown, we have:

$$\dot{E}_{in} - \cancel{\dot{E}_{out}} + \dot{E}_{gen} = \dot{E}_{st}$$

$$\dot{E}_{in} = q_e + q_s + q_{nw} \text{ where } q_e = q_{cond1} = k\left(\frac{\Delta y}{2}\times 1\right)\left(\frac{T_{m+1,n}^P - T_{m,n}^P}{\Delta x}\right) = \frac{k}{2}\left(T_{m+1,n}^P - T_{m,n}^P\right)$$

$$q_s = q_{cond2} = k\left(\Delta x \times 1\right)\left(\frac{T_{m,n-1}^P - T_{m,n}^P}{\Delta y}\right) = \frac{k}{2}\left(T_{m,n-1}^P - T_{m,n}^P\right) \text{ and } q_{nw} = q_{conv} = h\left(\frac{\Delta x}{\sqrt{2}}\times 1\right)\left(T_\infty^P - T_{m,n}^P\right)$$

$$\dot{E}_{gen} = \dot{q}V = \dot{q}\left[\left\{\left(\frac{1}{2}\times\frac{\Delta x}{2}\times\frac{\Delta y}{2}\right) + \left(\frac{\Delta x}{2}\times\frac{\Delta y}{2}\right)\right\}\times 1\right] = \frac{3}{8}\dot{q}\left(\Delta x\right)^2$$

$$\dot{E}_{st} = mC_p\frac{dT}{dt} = \rho V C_p\frac{dT}{dt} = \rho\left[\frac{3}{8}\left(\Delta x\right)^2\right]C_p\left(\frac{T_{m,n}^{P+1} - T_{m,n}^P}{\Delta t}\right)$$

Substituting in the energy balance, we have:

$$\frac{k}{2}\left(T_{m+1,n}^P - T_{m,n}^P\right) + k\left(T_{m,n-1}^P - T_{m,n}^P\right) + \frac{h\Delta x}{\sqrt{2}}\left(T_\infty^P - T_{m,n}^P\right) + \frac{3}{8}\dot{q}\left(\Delta x\right)^2 = \rho\left[\frac{3}{8}\left(\Delta x\right)^2\right]C_p\left(\frac{T_{m,n}^{P+1} - T_{m,n}^P}{\Delta t}\right)$$

Dividing throughout by $2/k$, we get:

$$\left(T_{m+1,n}^P - T_{m,n}^P\right) + 2\left(T_{m,n-1}^P - T_{m,n}^P\right) + \sqrt{2}\frac{h\Delta x}{k}\left(T_\infty^P - T_{m,n}^P\right) + \frac{3}{4}\frac{\dot{q}}{k}\left(\Delta x\right)^2 = \frac{3}{4}\frac{\rho C_p}{k}\frac{\left(\Delta x\right)^2}{\Delta t}\left(T_{m,n}^{P+1} - T_{m,n}^P\right)$$

where $Bi = \dfrac{h\Delta x}{k}$, $\alpha = \dfrac{k}{\rho C_p}$, and $Fo = \dfrac{\alpha \Delta t}{\left(\Delta x\right)^2}$

Name:______________________________
Last First

**Problem 2 - cont.**

Re-arrangement of the equation gives:

$$T_{m,n}^{P+1} = \frac{4}{3} Fo T_{m+1,n}^{P} + \frac{8}{3} Fo T_{m,n-1}^{P} + \frac{4\sqrt{2}}{3} Bi T_{\infty}^{P} + \frac{\dot{q}}{k} Fo (\Delta x)^2 + \left(1 - \frac{4\sqrt{2}}{3} Bi Fo - 4Fo\right) T_{m,n}^{P}$$

**Start your answer to part (b) here [10 pts]:**

For stability, we must have: $\left(1 - \frac{4\sqrt{2}}{3} Bi Fo - 4Fo\right) \geq 0$

i.e. $\left(1 + \frac{\sqrt{2}}{3} Bi\right) Fo \leq \frac{1}{4}$. This will give the required limitation on the time step to ensure stability in explicit method.

**Name:______________________________**
**Last First**

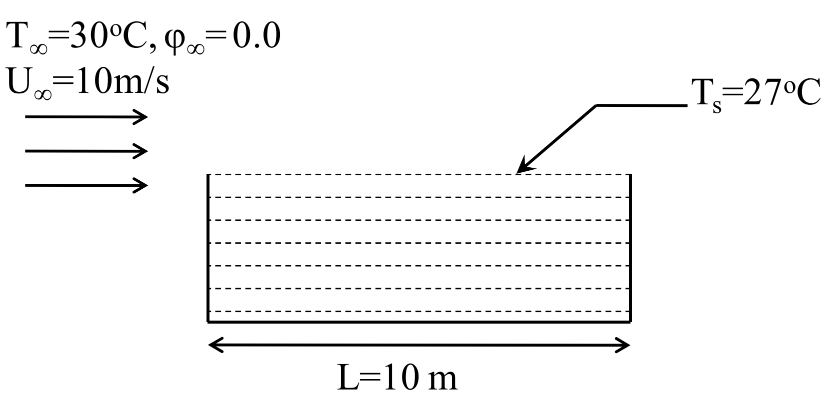
**Problem 3 [35 pts]**

An open swimming pool of area A = 10 m × 5 m is exposed to dry ambient air which flows with a velocity $u_\infty$ = 10 m/s at $T_\infty$ = 30°C. The conditions at the surface of the pool may be assumed to be saturated. Approximate this configuration as flow of air over a flat plate and use $\overline{Nu}_L = 0.037\,\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3}$ to calculate the Nusselt number. Consider that the water in the pool is to be maintained at a temperature of $T_s$ = 27°C and assume that the heat-mass transfer analogy is valid for the given conditions.
Assume the binary diffusion coefficient of water vapor in air is $D_{AB} = 0.26 \times 10^{-4}$ m²/s. All other properties should be obtained from the tables provided.

(a) Calculate the average heat transfer coefficient (W/m²-K) and the convective heat transfer rate (W).

(b) What is the rate of evaporation of water (kg/s) from the pool?

(c) Should the pool be refrigerated or heated in order to maintain its surface temperature at $T_s$ = 27°C? Calculate the external power (W) provided to/withdrawn from the pool.




**List your assumptions here [3 pts]:**
Steady state
Constant properties
Negligible radiation
Heat and mass transfer analogy valid
Boundary layer assumptions

**List the properties used and clearly mention the temperature for each property [5 pts]:**
All air properties at the film temperature: $T_{film} = 301.5$ K

$$\nu = 1.6\times10^{-5}\,\frac{\mathrm{m}^2}{\mathrm{s}};\; k = 0.026\,\frac{\mathrm{W}}{\mathrm{m\text{-}K}};\; Pr = \frac{\nu}{\alpha} = 0.7068;\; \alpha = 2.27\times10^{-5}\,\frac{\mathrm{m}^2}{\mathrm{s}}\;\text{(Table A-4)}$$

All water vapor properties at the surface temperature: $T_s = 300$ K

$$\rho_{sat,s} = \frac{1}{v_g} = \frac{1}{39.13}\,\frac{\mathrm{kg}}{\mathrm{m}^3};\; h_{fg} = 2438\,\frac{\mathrm{kJ}}{\mathrm{kg}}\;\text{(Table A-6)}$$

**Problem 3 - cont.**

Name:________________________________
Last First

**Start you answer to part (a) here [10 pts]:**

Reynolds number: $Re_L = \dfrac{u_\infty L}{\nu} = \dfrac{10\,\dfrac{\text{m}}{\text{s}} \times 10\ \text{m}}{1.6\times10^{-5}\,\dfrac{\text{m}^2}{\text{s}}} = 6.25\times10^6$

Average Nusselt number: $\overline{Nu_L} = 0.037 Re_L^{4/5} Pr^{1/3} = 9008.9 = \dfrac{\bar{h}L}{k}$

Average heat transfer coefficient: $\bar{h} = 23.4\,\dfrac{\text{W}}{\text{m}^2\text{-K}}$

Convective heat transfer rate from the ambient air to the pool surface:

$q_{conv} = \bar{h}A\left(T_\infty - T_s\right) = 22.6\,\dfrac{\text{W}}{\text{m}^2\text{-K}} \times (10\times5)\,\text{m}^2 \times (30-27)\,\text{K} \Rightarrow \underline{\boldsymbol{q_{conv} = 3{,}510\ \text{W}}}$

**Start you answer to part (b) here [7 pts]:**

Applying heat and mass transfer analogy: $\dfrac{\bar{h}}{\overline{h_m}} = \dfrac{k}{D_{AB} Le^{1/3}}$; $Le = \dfrac{\alpha}{D_{AB}} = \dfrac{2.27\times10^{-5}\,\dfrac{\text{m}^2}{\text{s}}}{2.6\times10^{-5}\,\dfrac{\text{m}^2}{\text{s}}} = 0.873$

Average mass transfer coefficient: $\overline{h_m} = 0.0224\,\dfrac{\text{m}}{\text{s}}$

Rate of evaporation of water vapor from the pool surface to the ambient air:

$\dot{m} = \overline{h_m}A\left(\rho_{A,s} - \rho_{A,\infty}\right) = \overline{h_m}A\left(\rho_{sat,s} - \cancel{\phi_\infty}\rho_{sat,\infty}\right) = 0.0224\,\dfrac{\text{m}}{\text{s}} \times (10\times5)\,\text{m}^2 \times (0.0256)\,\dfrac{\text{kg}}{\text{m}^3}$

$\Rightarrow \underline{\boldsymbol{\dot{m} = 0.0286\,\dfrac{\text{kg}}{\text{s}}}}$

**Start your answer to part (c) here [10 pts]:**

Heat transfer associated with evaporation of water vapor:

$q_{evap} = \dot{m}h_{fg} = 0.0286\,\dfrac{\text{kg}}{\text{s}} \times 2438\,\dfrac{\text{kJ}}{\text{kg}} \Rightarrow q_{evap} = 69{,}726.8\ \text{W}$

Considering energy balance at the pool surface, heat leaving the surface (due to evaporation) is higher than heat entering the surface (due to convection) $\Rightarrow$ the pool must be heated in order to maintain its surface at constant temperature of 27°C.

The external power provided to the pool: $P_e = q_{evap} - q_{conv} \Rightarrow \boldsymbol{P_e = 66{,}216.8\ \text{W}}$