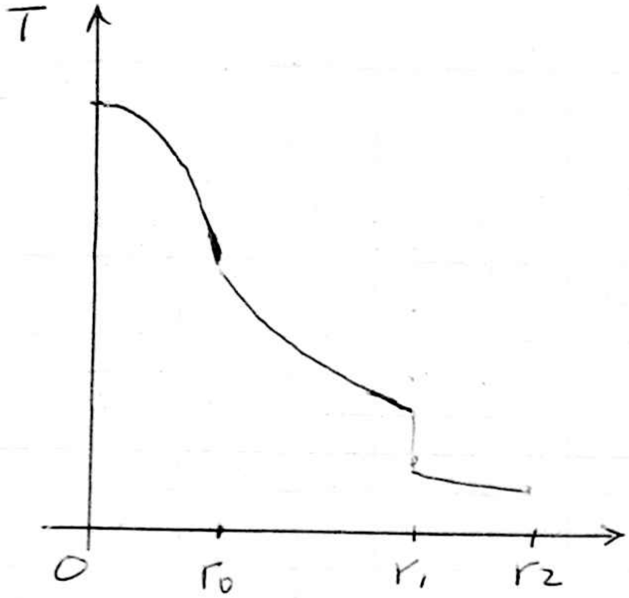
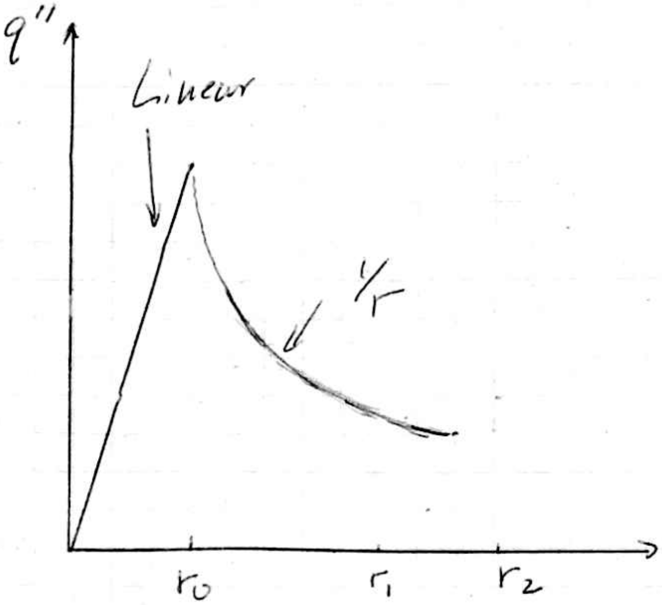
Prob 1





For inner rod: $T(r) = \frac{\dot{q} r_0^2}{4k}\left(1 - \frac{r^2}{r_0^2}\right)$

$$q'' = -k\frac{\partial T}{\partial r} = \frac{k\dot{q}\cdot 2r}{4k} = \frac{k\dot{q}r}{2k} \quad \text{linear.}$$

In the two layers: $T(r) = -\frac{\overset{\to 0}{\dot{q}}}{4k} r^2 + C_1 \ln r + C_2$

$$q'' = -k\frac{\partial T}{\partial r} = -k\frac{C_1}{r} \qquad \propto \frac{1}{r}$$

Key features:

T: at $r=0$, $dT/dr = 0$

at $r=r_0$, $-\cancel{k}\frac{dT}{dr}\Big|_- > -\cancel{k}\frac{dT}{dr}\Big|_+$

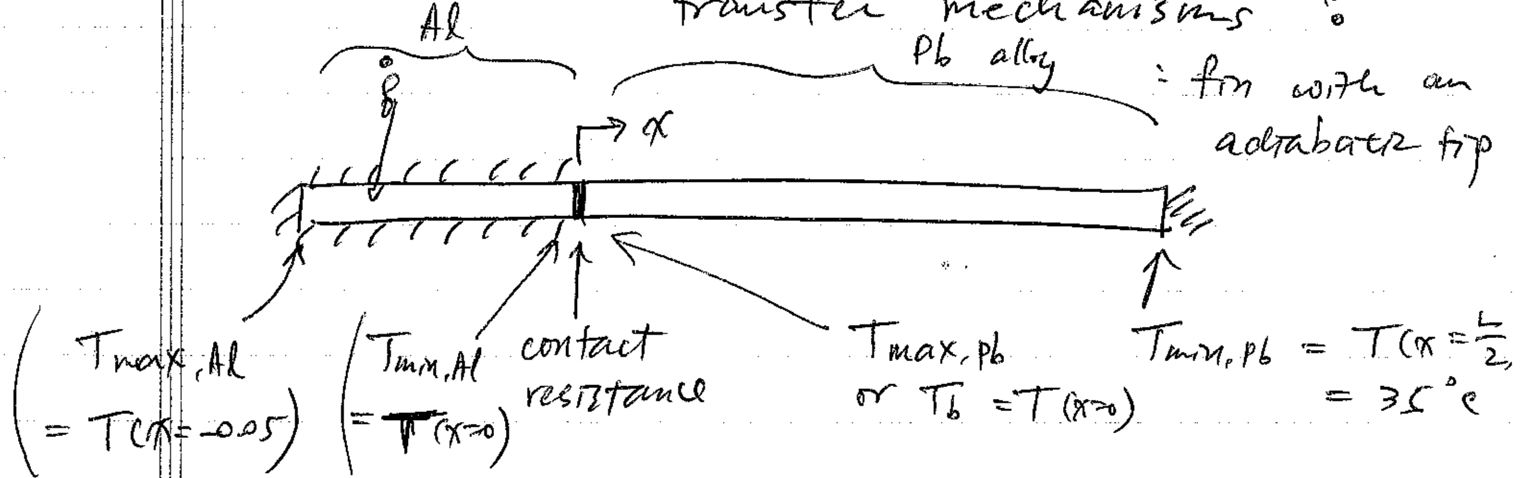
at $r=r_1$, $-\frac{dT}{dr}\Big|_- > -\frac{dT}{dr}\Big|_+$

at $r=r_1$, there is a temperature drop.

q'': Linear in rod. $\frac{1}{r}$ in two layers.

Prob 2

Given the symmetry in the problem, we find heat transfer mechanisms :



① Fin : $\dfrac{\theta(x)}{\theta_b} = \dfrac{\cosh\left[m\left(\frac{L}{2}-x\right)\right]}{\cosh(mL/2)} \Rightarrow T_b = 76.4\,^\circ C$

at $x=0$: $q_b = q_{contact} = q_{fin}$

$q_b = \dot{q}V = q_{fin} = \sqrt{hPkA_c}\;\theta_b \tanh\left(m\frac{L}{2}\right)$

$\Rightarrow \dot{q} = 1.09 \times 10^6\ W/m^3$

② $q''_{contact} = \dfrac{T_{min\,Al} - T_{max,Pb}}{R''_{t,contact}} = \dfrac{T_{min,Al} - 76.4\,^\circ C}{2\times10^{-4}\ \frac{m^2 K}{W}}$

$\Rightarrow T_{min\,Al} = 87.3\,^\circ C$

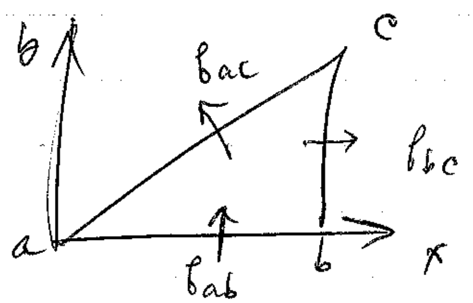
③ In Al rod, $T(x) = -\dfrac{\dot{q}x^2}{2k} + C_1 x + C_2$

BCs) $T(x=0) = 87.3\,^\circ C$ and $\left.\dfrac{dT}{dx}\right|_{x=-0.05m} = 0$

Thus, C_1 and C_2 can be determined.

$T_{max,Al} = T(x=-0.05\,m) = 92.9\,^\circ C$

Prob. 3.



$T(x,y)$ given
$= 100(x^2+y^2)+500\,K$

① $q_{ab} = -k \left.\dfrac{\partial T}{\partial y}\right|_{y=0} (\Delta x \cdot 1) = 0\ W$

② $q_{bc} = -k \left.\dfrac{\partial T}{\partial x}\right|_{x=1} (\Delta y \cdot 1) = -200\ W$

③ Energy balance

$$\cancel{\rho c \frac{\partial T}{\partial t}} = k\frac{\partial^2 T}{\partial x^2} + k\frac{\partial^2 T}{\partial y^2} + \cancel{k\frac{\partial^2 T}{\partial z^2}} + \dot{q}''$$

$$\dot{q}''' = -k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = -400\ W/m^3$$

④ $\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$

$$0 = q_{ab} - q_{bc} - q_{ac} + \dot{q}'''V$$

$$q_{ac} = 0\ W$$

Also considering symmetry in Temp profile, you can get $q_{ab} = q_{ac} = 0\ W$ automatically.

Prob. 4.

Assumptions: neglect radiation, lumped capacitance method to be evaluated

a, Grape.

$Bi = \frac{h L_c}{k} = 0.0083 < 0.1$, using L.C.M.

$t = -\frac{\rho C_p V}{h A_s} \ln \frac{\theta}{\theta_i} = \cdots = 2575$ (s)

[If using analytical solution, $Bi = 0.025$

$\zeta_1 = 0.2718$, $C_1 = 1.0075$,

$t = -\frac{r^2}{\zeta^2} \frac{\rho C_p}{k} \ln \frac{\theta_o^*}{C_1} = 2626$ (s), 2%]

b, Watermelon;

$Bi = \frac{h L_c}{k} = 0.267 > 0.1$, so using analytical soln

To use Table 5.1, $Bi = \frac{h r}{k} = 0.8$

From Table 5-1, $\zeta_1 = 1.4320$, $C_1 = 1.2236$

$t = -\frac{r^2}{\zeta^2} \frac{\rho C_p}{k} \ln \frac{\theta_o^*}{C_1} = 47484$ (s)

[Using L.C.M will produce $t = 36051$ (s),

a 32% error]