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## Circle one:

# School of Mechanical Engineering <br> Purdue University ME315 Heat and Mass Transfer 

## Exam \#1

February 20, 2014

## Instructions:

- Write your name on each page
- Write on one side of the page only
- Keep all the pages in order
- You are asked to write your assumptions and answers to sub-problems in designated areas. Only work in its designated area will be graded.

| Performance |  |  |
| :---: | :---: | :---: |
| 1 25 <br> 2 25 <br> 3 25 <br> 4 25 <br> Total 100 |  |  | |  |
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## Problem 1 (25 points)

Electric current flows through a long conducting rod with a radius $r_{0}$, generating thermal energy at a uniform volumetric rate of $\dot{q}$. There are two layers surrounding this conducting rod as shown in the figure below, with radius $r_{1}$ and $r_{2}$. The thermal conductivity of the rod and the two surrounding layers are: $k_{0}<k_{1}<k_{2}$ (see figure). Contact resistance exists between the two outer layers, but not between the rod and layer 1. Convection cooling occurs at the out-most surface.

(a) Sketch the steady-state temperature distribution in the rod and the two outer layers as a function of $r$ ( $r=0$ at the center of the rod) in the figure given below ( 10 points).
(b) Sketch the steady-state distribution of heat flux $q$ " $\left[\mathrm{W} / \mathrm{m}^{2}\right]$ in the conductor and the two outer layers as a function of $r$ in the figure given below (10 points).
(c) Identify key features in your sketch of the temperature distribution in (a) and heat flux distribution (b), and explain briefly the reason for these features (5 points).



Name

Prob. 1 - cont'd
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## Problem 2 (25 points)

Consider a circular rod (diameter $d=10 \mathrm{~mm}$, length $\mathrm{L}=100 \mathrm{~mm}$ ) made of lead alloy is attached to two circular aluminum rods of same diameter (both diameters $d=10 \mathrm{~mm}, l=50 \mathrm{~mm}$ ) at its two ends. The aluminum rods are embedded in two walls as shown, but the lead alloy rod is exposed to an airstream at conditions $h=100 \mathrm{~W} / \mathrm{m}^{2} \cdot \mathrm{~K}$ and a free stream temperature $\mathrm{T}_{\infty}=20^{\circ} \mathrm{C}$. The contacts at the two joints between the rods are not perfect, with a contact resistance $2 \times 10^{-4}$ $\mathrm{m}^{2} \cdot \mathrm{~K} / \mathrm{W}$. An electromagnetic field induces volumetric energy generation at a uniform rate $\dot{q}$ within the embedded aluminum rods. The wall is made of perfectly insulating materials ( $k_{\mathrm{w}}=0$ $\mathrm{w} / \mathrm{mK}$ ).
(a) Find the uniform heat generation rate $\dot{q}$ in the embedded aluminum rods so that the lowest temperature in the lead alloy rod is maintained at $35^{\circ} \mathrm{C}$.
(b) For this condition, what is the lowest temperature in the aluminum rod?
(c) What is the maximum temperature in the aluminum rod?

Thermal conductivity of the aluminum rod is $240 \mathrm{~W} / \mathrm{m} \cdot \mathrm{K}$, and that of lead alloy is $25 \mathrm{~W} / \mathrm{m} \cdot \mathrm{K}$.


## List assumptions here (2 points)

## Start part (a) here (13 points)

Prob. 2 - cont'd

## Start part (b) here (6 points)

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## Problem 3 [25 points]

Consider 2D steady-state heat conduction in a triangular wedge with constant properties. The thermal conductivity of the wedge is $1 \mathrm{~W} / \mathrm{m} \cdot \mathrm{K}$. The coordinates of the wedge are shown in meters. The temperature distribution in the wedge is described by $T(x, y)=100\left(x^{2}+y^{2}\right)+500$ K.

(a) Compute the heat transfer rates, $q_{a b}$ and $q_{b c}$, in Watts per unit depth of the wedge, which are normal to the faces, $a b$ and $b c$, respectively. (15 points)
(b) Determine the volumetric heat generation rate, $\dot{q}^{\prime \prime \prime}$, in $\mathrm{W} / \mathrm{m}^{3}$ and the heat transfer rate, $q_{a c}$, in Watts per unit depth. (10 points) Hint: consider possible symmetry in the temperature distribution.
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## Problem 4 [25 points]

Consider a grape and a watermelon that are both initially at $30^{\circ} \mathrm{C}$. At time $t=0$ they are placed in a refrigerator whose inside air is at $5^{\circ} \mathrm{C}$ and provides a convection heat transfer coefficient $h=5$ $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$. The grape and the watermelon can be approximated as spheres with diameter $D=20$ mm and 320 mm , respectively. The thermophysical properties of the grape and the watermelon are approximated as follows:
Grape: $\rho=1,200 \mathrm{~kg} / \mathrm{m}^{3}, k=2 \mathrm{~W} / \mathrm{mK}$, and $c_{\mathrm{p}}=2,000 \mathrm{~J} / \mathrm{kg}$;
Watermelon: $\rho=700 \mathrm{~kg} / \mathrm{m}^{3}, k=1 \mathrm{~W} / \mathrm{mK}$, and $c_{\mathrm{p}}=3,000 \mathrm{~J} / \mathrm{kg}$.
Estimate the time $t$ (in seconds) required for the centers of (a) the grape and (b) the watermelon to cool from $30^{\circ} \mathrm{C}$ to $10^{\circ} \mathrm{C}$.

## List assumptions here (2 pts)

Start part (a), the grape, here (11 points)

Problem 4 - cont'd

Start part (b), the watermelon, here (12 points)

