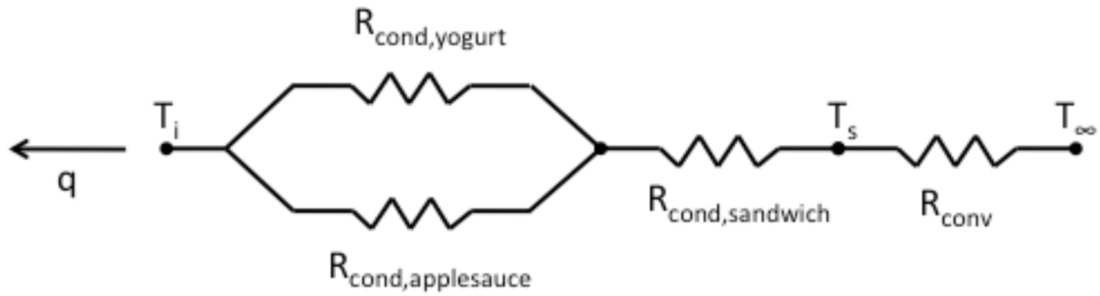
Problem 1

(a)

Assuming the interfacial temperature between sandwich and yogurt is the same as that of between sandwich and applesauce,



$$R_{cond,yogurt} = \frac{H_a}{K_y \cdot A_y} = \frac{H_a}{K_y \cdot \left(\frac{L}{2}\right) \cdot D}$$

$$R_{cond,applesauce} = \frac{H_a}{K_a \cdot A_a} = \frac{H_a}{K_a \cdot \left(\frac{L}{2}\right) \cdot D}$$

$$\frac{1}{R_{sum,parallel}} = \frac{1}{R_{cond,yogurt}} + \frac{1}{R_{cond,applesauce}} = \frac{K_y \cdot \left(\frac{L}{2}\right) \cdot D}{H_a} + \frac{K_a \cdot \left(\frac{L}{2}\right) \cdot D}{H_a} = \frac{\left(K_y + K_a\right) \cdot \left(\frac{L}{2}\right) \cdot D}{H_a}$$

$$\therefore R_{sum,parallel} = \frac{H_a}{\left(K_y + K_a\right) \cdot \left(\frac{L}{2}\right) \cdot D}$$

$$R_{cond,sandwich} = \frac{H_s}{K_s \cdot A_s} = \frac{H_s}{K_s \cdot L \cdot D}$$

$$R_{conv} = \frac{1}{h \cdot A_s} = \frac{1}{h \cdot L \cdot D}$$

$$R_{tot} = R_{sum,parallel} + R_{cond,sandwich} + R_{conv}$$

$$\boxed{\therefore R_{tot} = \frac{H_a}{\left(K_y + K_a\right) \cdot \left(\frac{L}{2}\right) \cdot D} + \frac{H_s}{K_s \cdot L \cdot D} + \frac{1}{h \cdot L \cdot D} = \frac{1}{L \cdot D} \cdot \left[\frac{2 \cdot H_a}{\left(K_y + K_a\right)} + \frac{H_s}{K_s} + \frac{1}{h}\right]}$$

(b)

$$q = \dot{q} \cdot V_{icepack} = \dot{q} \cdot L \cdot H_i \cdot D = \left(1.2\ kW/m^2\right) \cdot (0.25\ m) \cdot \left(0.03\ m\right) \cdot (0.25\ m)$$

$$\boxed{\therefore q = 2.25\ W}$$

(c)

$$q = \frac{T_\infty - T_s}{R_{conv}}; \qquad R_{conv} = \frac{1}{h \cdot A_s} = \frac{1}{h \cdot L \cdot D} = \frac{1}{\left(10\ W/m^2 - K\right) \cdot \left(0.25\ m\right)^2} = 1.6\ K/W$$

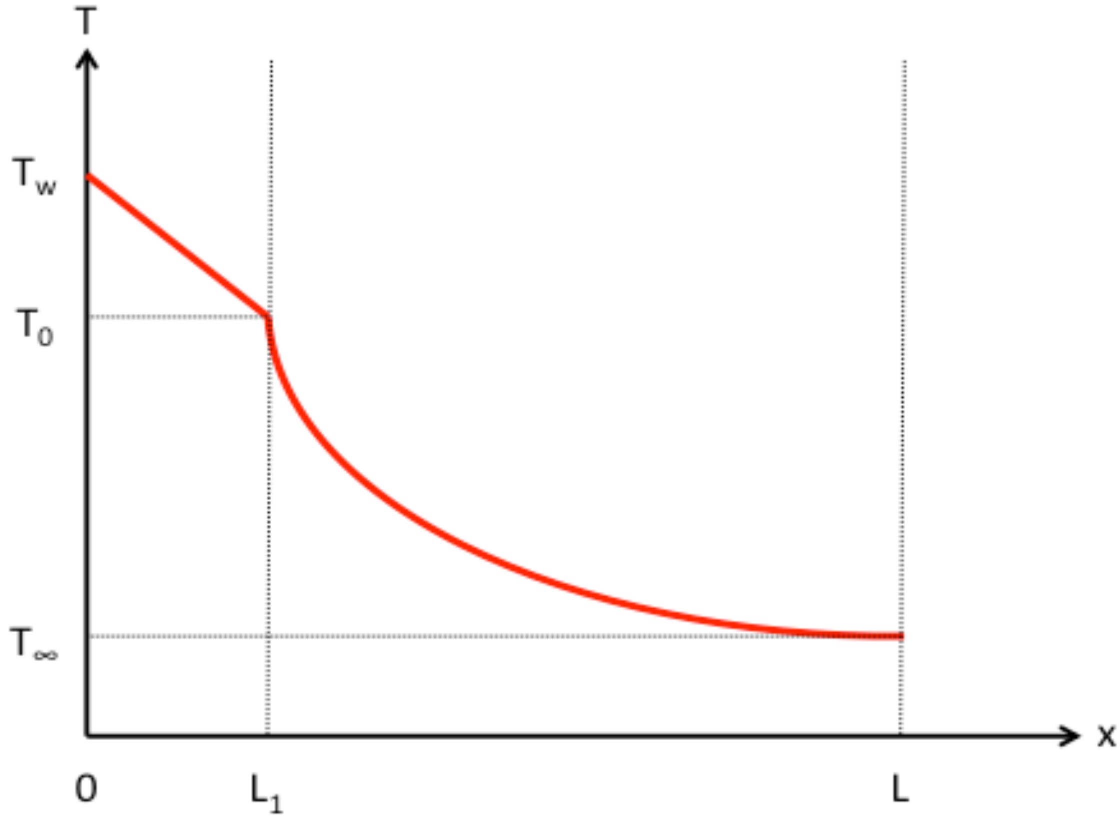
$$q = \frac{T_\infty - T_s}{R_{conv}} = \frac{(25°C - T_s)}{1.6\ K/W} = 2.25\ W$$

$$T_s = 25°C - \left(2.25\ W\right) \cdot \left(1.6\ K/W\right)$$

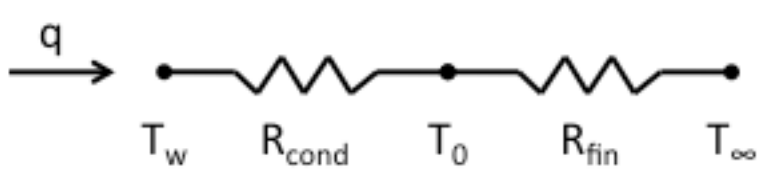
$$\boxed{\therefore T_s = 21.4°C}$$

Problem 2

(a)



(b)



$$R_{cond} = \frac{L_1}{k \cdot A_c} = \frac{L_1}{k \cdot \frac{\pi D^2}{4}} = \frac{(0.2\, m)}{(100\, W/m-K) \cdot \frac{\pi (0.025\, m)^2}{4}} = 4.07\, K/W$$

$$q_{fin} = \left(h\ P\ k\ A_c\right)^{\frac{1}{2}} \cdot \theta_b = \left(h\ P\ k\ A_c\right)^{\frac{1}{2}} \cdot \left(T_0 - T_\infty\right) = \frac{\left(T_0 - T_\infty\right)}{\left(h\ P\ k\ A_c\right)^{-\frac{1}{2}}} = \frac{\left(T_0 - T_\infty\right)}{R_{fin}}$$

where

$$P = \pi D \quad and \quad A_c = \frac{\pi D^2}{4}$$

$$\therefore R_{fin} = \left(h\ P\ k\ A_c\right)^{-\frac{1}{2}} = \left[\left(20\ W/m^2 - K\right)\pi\left(0.025\ m\right)\left(100\ W/m - K\right)\frac{\pi\left(0.025\ m\right)^2}{4}\right]^{-\frac{1}{2}}$$

$$= 3.6013\ K/W$$

$$q = q'' \cdot A_c = (45 \times 10^3\ W/m^2) \cdot \frac{\pi\left(0.025\ m\right)^2}{4} = 22.089\ W$$

$$q = \frac{T_0 - T_\infty}{R_{fin}} = \frac{T_0 - 25°C}{3.6013\ K/W} = 22.089\ W$$

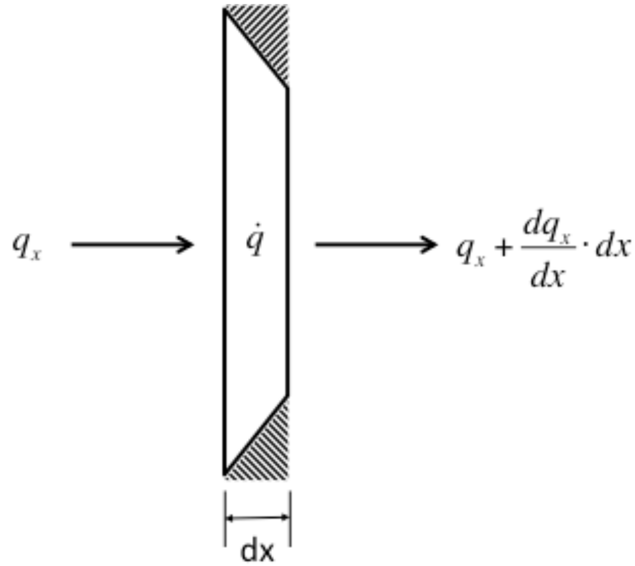
$$\boxed{\therefore T_0 = 104.55°C}$$

$$q = \frac{T_w - T_0}{R_{cond}} = \frac{T_w - 104.55°C}{4.07\ K/W} = 22.089\ W$$

$$\boxed{\therefore T_w = 194.45°C}$$

Problem 3

(a)



Assumptions:

1. 1-D conduction
2. Steady state
3. Constant properties

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{store}$$

$$\dot{E}_{in} = q_x, \qquad \dot{E}_{out} = q_x + \frac{dq_x}{dx} \cdot dx, \qquad \dot{E}_{gen} = \dot{q} \cdot A(x) \cdot dx = \dot{q} \cdot A_0 e^{-ax} \cdot dx$$

$$\dot{E}_{store} = 0 \quad (Steady\ state)$$

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = q_x - \left(q_x + \frac{dq_x}{dx} \cdot dx\right) + \dot{q} \cdot A_0 e^{-ax} \cdot dx = 0$$

$$\boxed{\therefore q_x - \left(q_x + \frac{dq_x}{dx} \cdot dx\right) + \dot{q} \cdot A_0 e^{-ax} \cdot dx = 0}$$

(b)

$$q_x - \left(q_x + \frac{dq_x}{dx} \cdot dx \right) + \dot{q} \cdot A_0 e^{-ax} \cdot dx = 0$$

$$-\frac{dq_x}{dx} \cdot dx + \dot{q} \cdot A_0 e^{-ax} \cdot dx = -\frac{dq_x}{dx} + \dot{q} \cdot A_0 e^{-ax} = 0$$

$$q_x = -k \cdot \frac{dT}{dx} \cdot A(x) = -k \cdot \frac{dT}{dx} \cdot A_0 e^{-ax}$$

$$\therefore -\frac{dq_x}{dx} + \dot{q} \cdot A_0 e^{-ax} = -\frac{d}{dx}\left(-k \cdot \frac{dT}{dx} \cdot A_0 e^{-ax} \right) + \dot{q} \cdot A_0 e^{-ax}$$

$$= k \cdot \frac{d}{dx}\left(\frac{dT}{dx} \cdot e^{-ax} \right) + \dot{q} \cdot e^{-ax} = \frac{d}{dx}\left(\frac{dT}{dx} \cdot e^{-ax} \right) + \frac{\dot{q}}{k} \cdot e^{-ax}$$

$$= \frac{d^2T}{dx^2} \cdot e^{-ax} - a \cdot e^{-ax} \cdot \frac{dT}{dx} + \frac{\dot{q}}{k} \cdot e^{-ax}$$

$$= \frac{d^2T}{dx^2} - a \cdot \frac{dT}{dx} + \frac{\dot{q}}{k} = 0$$

$$\boxed{\therefore \frac{d^2T}{dx^2} - a \cdot \frac{dT}{dx} + \frac{\dot{q}}{k} = 0}$$

$$BC1: \quad T_{x=0} = T_0$$

$$BC2: \quad -k \left. \frac{dT}{dx} \right|_{x=L} \cdot A_{x=L} = h \cdot (T_{x=L} - T_\infty) \cdot A_{x=L}$$

$$\therefore -k \left. \frac{dT}{dx} \right|_{x=L} = h \cdot (T_{x=L} - T_\infty)$$