ME 315
Exam 1 Solutions
8:00 -9:00 PM
Tuesday, February 10, 2009

1. (25 points) Consider a composite slab composed of a layer of plastic bonded to a layer of copper. The thickness of the plastic layer is *15 mm* while that of the copper layer is *10 mm*. The cross-sectional area of the slab is $100\ cm^2$. There is contact resistance at the plastic-Cu interface due to the bonding. During testing, the plastic side is subjected to a hot fluid stream at $T_{\infty 1} = 400\ K$ and a heat transfer coefficient of $200\ W/m^2K$. The copper side is subjected to surrounding air at temperature $T_{\infty 2} = 300\ K$, with a heat transfer coefficient of $100\ W/m^2K$.

You are also given the following data:
Thermal contact conductance at plastic-Cu interface = $420\ W/m^2K$.
$k_{plastic} = 2\ W/mK$; $k_{cu} = 400\ W/mK$

(a) Draw a thermal circuit for the composite showing all relevant thermal resistances and their numerical values.

<u>Solution</u>

$$L_{plastic} = t_{plastic} = 15mm = 0.015m$$

$$L_{Cu} = t_{Cu} = 10mm = 0.010m$$

$$A_c = 100cm^2 \frac{(1m)^2}{(100cm)^2} = 0.01m^2$$

$$\therefore R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(200 W/m^2K)(0.01m^2)} = \boxed{0.5 K/W}$$

$$R_{cond,P} = \frac{L_P}{k_P A} = \frac{0.015m}{(2 W/mK)(0.01m^2)} = \boxed{0.75 K/W}$$

$$R_{contact} = \frac{1}{k_{contact} A} = \frac{1}{\left(420 W/m^2 K\right)\left(0.01 m^2\right)} = \boxed{0.2381 K/W}$$

$$R_{cond,Cu} = \frac{L_{Cu}}{k_{Cu} A} = \frac{0.010 m}{\left(400 W/mK\right)\left(0.01 m^2\right)} = \boxed{0.0025 K/W}$$

$$R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{\left(100 W/m^2 K\right)\left(0.01 m^2\right)} = \boxed{0.1 K/W}$$

(b) What is the drop in temperature across the plastic-Cu interface in *K* (i.e. the temperature drop across the contact resistance)?

Solution

$$R_{tot} = 0.5 + 0.75 + 0.2381 + 0.0025 + 0.1$$

$$= 1.5906 K/W$$

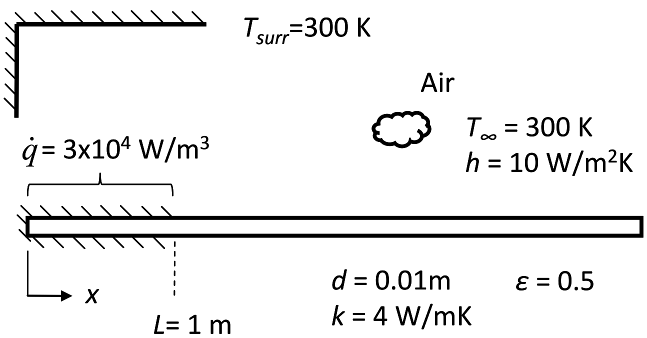
$\Delta T_{interface}$ = 14.95268 K

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} = \frac{400 - 300}{1.5906} = 62.8 W$$

$$\Delta T_{interface} = R_{contact} \cdot q$$

$$= 0.2381 \cdot 62.8 = \boxed{14.95268 K}$$

2. (35 Points) A very long circular rod of *0.01m* diameter and thermal conductivity $k = 4\ W/mK$ is placed in a large enclosure. A small portion of the rod ($0 \le x \le L$) is perfectly insulated and experiences uniform heat generation, $\dot{q} = 3\times10^4$ W/m^3. The rod loses heat by convection and radiation, as shown. The rod has a gray surface with emissivity $\varepsilon = 0.5$ and is located in surroundings at $T_{surr} = 300K$. The convective heat transfer coefficient is $h = 10\ W/m^2K$, and the environment temperature is also *300K*.



(i) Determine the heat flux q''_b at $x=L$ in W/m^2.

<u>Solution</u>

$$\dot{q}\cdot V = \left(3\times10^4\ W/m^2\right)\cdot\frac{\pi D^2}{4}\cdot L$$

$$= \left(3\times10^4\ W/m^2\right)\cdot\frac{\pi(0.01m)^2}{4}\cdot(1m) = 2.3562W$$

$$= q$$

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \rightarrow \therefore \dot{E}_{gen} = \dot{E}_{out}$$

$$\therefore q''_b = \frac{q}{A} = \frac{q}{\frac{\pi D^2}{4}} = \frac{2.3562W}{\frac{\pi(0.01m)^2}{4}} = \boxed{3\times10^4\ W/m^2}$$

(ii) Find the effective heat transfer coefficient h_{rad} in W/m^2K. Evaluate it at an approximate surface temperature $T_s = 400\ K$. You may use this quantity in the analysis for part (iii).

Solution

$$h_{rad} = \varepsilon \cdot \sigma \cdot (T_s + T_{Surr}) \cdot (T_s^2 + T_{Surr}^2)$$

$$T_s = 400K$$

$$T_{surr} = T_\infty = 300K \qquad \varepsilon = 0.5$$

$$\therefore h_{rad} = (0.5) \cdot (5.67 \times 10^{-8}\ W/m^2K^4) \cdot (400 + 300)K \cdot (400^2 + 300^2)K^2$$

$$= \boxed{4.96125 W/m^2K}$$

(iii) Find the temperature T_b at $x=L$ in ^oK.

Solution

Infinite fin $q_f = M = \sqrt{h_{tot} PkA_c}(T_b - T_\infty);\ \ h_{tot} = h_{rad} + h_{conv}$

$$\dot{q} \cdot V = q_t$$

$$\dot{q} \cdot A_c \cdot L = \sqrt{h_{tot} PkA_c}(T_b - T_\infty)$$

$$\therefore T_b - T_\infty = \frac{1}{\sqrt{h_{tot} PkA_c}} \cdot \dot{q} \cdot A_c \cdot L$$

$$\Rightarrow T_b = T_\infty + \frac{1}{\sqrt{h_{tot} PkA_c}} \cdot \dot{q} \cdot A_c \cdot L$$

$$A_c = \frac{\pi D^2}{4} = 0.000079 m^2 \qquad P = \pi D = 0.031416 m$$

$$\therefore T_b = 300K + \frac{3 \times 10^4\ W/m^3 \cdot (0.000079 m^2) \cdot 1m}{\sqrt{(10 + 4.96125) W/m^2K \cdot (0.031416 m) \cdot (4 W/mK) \cdot (0.000079 m^2)}}$$

$$= 300K + \frac{2.37W}{0.012187 W/K} = \boxed{494.47K}$$

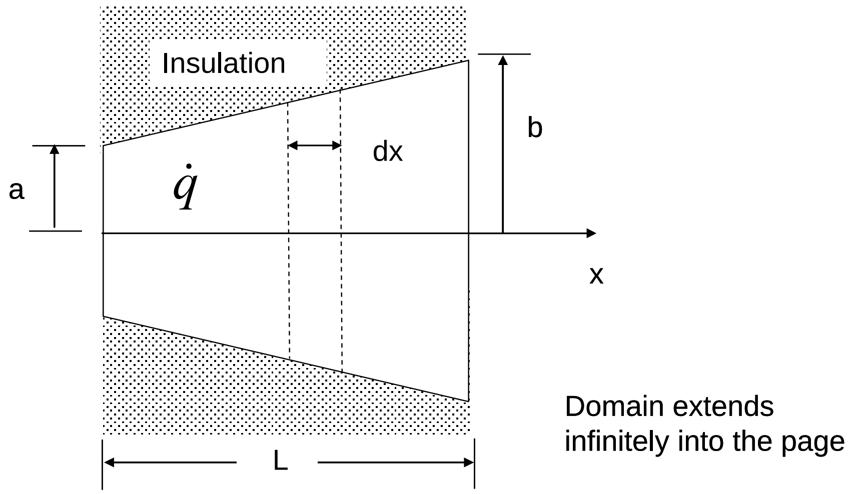
(iv) Find the fin effectiveness ε_f.

Solution

$$\varepsilon_f = \frac{q_{tin}}{h_{tot} \cdot A_{c,b} \cdot \theta_b} = \frac{\sqrt{h_{tot} P k A_c}\,\theta_b}{h_{tot} \cdot A_c \cdot \theta_b} = \frac{\sqrt{Pk}}{\sqrt{h_{tot} \cdot A_c}}$$

$$= \frac{\sqrt{(0.031416m)\cdot(4W/mK)}}{\sqrt{(14.96125W/m^2K)\cdot(0.000079m^2)}} = \boxed{10.31}$$

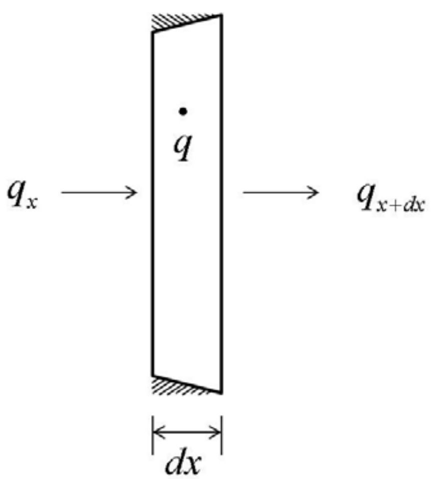
3. (40 Points) Consider steady 1D heat conduction in the trapezoidal domain shown below. The domain extends infinitely into the page. There is a constant volumetric heat generation rate $\dot{q}$ in the domain.



The half-width of the domain at $x=0$ is a, and that at $x=L$ is b as shown. The temperature at $x=0$ is T_0, while that at $x=L$ is T_L. The thermal conductivity of the material is k and may be assumed constant.

(i) Consider an infinitesimal control volume of extent dx as shown. Write an energy balance for the control volume to develop a differential equation for the heat transfer rate $q_x(x)$. State all assumptions clearly.

Solution



$$R = \frac{b-a}{L}x + a$$

$$A(x) = 2\left(a + \frac{b-a}{L}x\right)$$

$$\frac{dA(x)}{dx} = \frac{2(b-a)}{L}$$

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$\dot{E}_{in} = q_x$$

$$\dot{E}_{out} = q_x + \frac{dq_x}{dx}dx$$

$$\dot{E}_{gen} = \dot{q}\, A(x) dx$$

$$q_x - q_x - \frac{dq_x}{dx} dx + \dot{q}\, A(x) dx = 0$$

$$-\frac{dq_x}{dx} + \dot{q}\, A(x) = 0$$

(ii) Now, using Fourier's law, develop a differential equation for the temperature *T(x)*.

Solution

$$q_x = -k \frac{dT}{dx} \cdot A(x)$$

$$\therefore \frac{d}{dx}\left(-k \frac{dT}{dx} \cdot A(x) \right) = \dot{q}\, A(x)$$

$$-k \frac{d}{dx}\left[\frac{dT}{dx} \cdot A(x) \right] = -k\left[\frac{d^2T}{dx} \cdot A(x) + \frac{dT}{dx} \cdot \frac{dA(x)}{dx} \right]$$

$$\therefore \frac{d^2T}{dx} \cdot A(x) + \frac{dT}{dx} \cdot \frac{dA(x)}{dx} = -\frac{\dot{q}}{k} A(x)$$

$$A(x) \frac{d^2T}{dx^2} + \frac{dA(x)}{dx} \frac{dT}{dx} = -\frac{\dot{q}}{k} A(x)$$

$$2\left(a + \frac{b-a}{L} x \right) \frac{d^2T}{dx^2} + 2\left(\frac{b-a}{L} \right) \frac{dT}{dx} + 2\left(a + \frac{b-a}{L} x \right) \frac{\dot{q}}{k} = 0$$

(iii) Write down the boundary conditions necessary to solve the *T(x)* differential equation. DO NOT SOLVE THE EQUATION.

Solution

$$T(x = 0) = T_0$$

$$T(x = L) = T_L$$