ME 315
Exam 1
8:00 -9:00 PM
Tuesday, February 10, 2009

- This is a closed-book, closed-notes examination. *There is a formula sheet at the back.*
- You must turn off all communications devices before starting this exam, and leave them off for the entire exam.
- Please write legibly and show all work for your own benefit. Please show your final answers in the boxes provided.
- State all assumptions.
- Please arrange all your sheets in the correct order. Make sure they are all included.

Name: ____________________________________
Last First

CIRCLE YOUR DIVISION

Div. 1 (9:30 am)
Prof. Murthy

Div. 2 (12:30 pm)
Prof. Choi

Problem	Score
1 (25 Points)	
2 (35 Points)	
3 (40 Points)	
Total (100 Points)	

1. (25 points) Consider a composite slab composed of a layer of plastic bonded to a layer of copper. The thickness of the plastic layer is *15 mm* while that of the copper layer is *10 mm*. The cross-sectional area of the slab is $100\ cm^2$. There is contact resistance at the plastic-Cu interface due to the bonding. During testing, the plastic side is subjected to a hot fluid stream at $T_{\infty 1} = 400\ K$ and a heat transfer coefficient of $200\ W/m^2K$. The copper side is subjected to surrounding air at temperature $T_{\infty 2} = 300\ K$, with a heat transfer coefficient of $100\ W/m^2K$.

You are also given the following data:
Thermal contact conductance at plastic-Cu interface = $420\ W/m^2K$.
$k_{plastic} = 2\ W/mK$; $k_{cu} = 400\ W/mK$

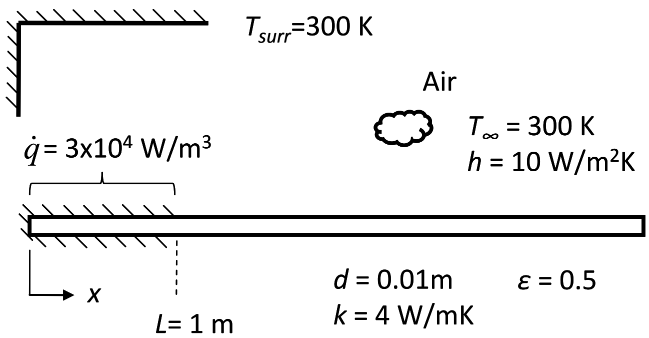
(a) Draw a thermal circuit for the composite showing all relevant thermal resistances and their numerical values.

● $T_{\infty 1}$ ● $T_{\infty 2}$

(b) What is the drop in temperature across the plastic-Cu interface in *K* (i.e. the temperature drop across the contact resistance)?

$\Delta T_{interface}$ = K

2. (35 Points) A very long circular rod of *0.01m* diameter and thermal conductivity $k = 4\ W/mK$ is placed in a large enclosure. A small portion of the rod ($0 \leq x \leq L$) is perfectly insulated and experiences uniform heat generation, $\dot{q} = 3\times10^4$ W/m^3. The rod loses heat by convection and radiation, as shown. The rod has a gray surface with emissivity $\varepsilon = 0.5$ and is located in surroundings at $T_{surr} = 300K$. The convective heat transfer coefficient is $h=10\ W/m^2K$, and the environment temperature is also *300K*.



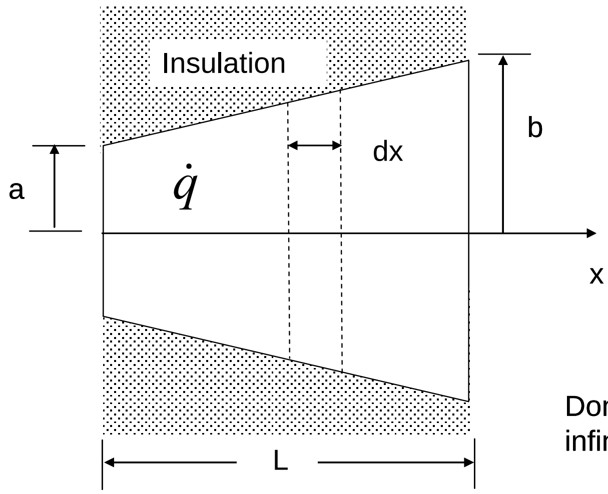
(i) Determine the heat flux q''_b at $x=L$ in W/m^2.

(ii) Find the effective heat transfer coefficient h_{rad} in W/m^2K. Evaluate it at an approximate surface temperature $T_s = 400\ K$. You may use this quantity in the analysis for part (iii).

(iii) Find the temperature T_b at $x=L$ in ^oK.

(iv) Find the fin effectiveness ε_f.

3. (40 Points) Consider steady 1D heat conduction in the trapezoidal domain shown below. The domain extends infinitely into the page. There is a constant volumetric heat generation rate $\dot{q}$ in the domain.



Domain extends infinitely into the page

The half-width of the domain at $x=0$ is a, and that at $x=L$ is b as shown. The temperature at $x=0$ is T_0, while that at $x=L$ is T_L. The thermal conductivity of the material is k and may be assumed constant.

(i) Consider an infinitesimal control volume of extent dx as shown. Write an energy balance for the control volume to develop a differential equation for the heat transfer rate $q_x(x)$. State all assumptions clearly.

(ii) Now, using Fourier's law, develop a differential equation for the temperature $T(x)$.

(iii) Write down the boundary conditions necessary to solve the $T(x)$ differential equation. DO NOT SOLVE THE EQUATION.

BASIC EQUATION SHEET

Conservation Laws

Control Volume Energy Balance: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$; $\dot{E}_{st} = mC_p \, {dT}/{dt}$; $\dot{E}_{gen} = \dot{q} V$

Surface Energy Balance: $\dot{E}_{in} - \dot{E}_{out} = 0$

Conduction

Fourier's Law: $q''_{cond,x} = -k \dfrac{\partial T}{\partial x}$; $q''_{cond,n} = -k \dfrac{\partial T}{\partial n}$; $q_{cond} = q''_{cond} A$

Heat Flux Vector: $\overrightarrow{q''} = q''_x \vec{i} + q''_y \vec{j} + q''_z \vec{k} = -k \left[\dfrac{\partial T}{\partial x} \vec{i} + \dfrac{\partial T}{\partial y} \vec{j} + \dfrac{\partial T}{\partial z} \vec{k} \right]$

Heat Diffusion Equation:

Rectangular Coordinates: $\dfrac{\partial}{\partial x}\left(k \dfrac{\partial T}{\partial x} \right) + \dfrac{\partial}{\partial y}\left(k \dfrac{\partial T}{\partial y} \right) + \dfrac{\partial}{\partial z}\left(k \dfrac{\partial T}{\partial z} \right) + \dot{q} = \rho C_p \dfrac{\partial T}{\partial t}$

Cylindrical Coordinates: $\dfrac{1}{r} \dfrac{\partial}{\partial r}\left(kr \dfrac{\partial T}{\partial r} \right) + \dfrac{1}{r^2} \dfrac{\partial}{\partial \phi}\left(k \dfrac{\partial T}{\partial \phi} \right) + \dfrac{\partial}{\partial z}\left(k \dfrac{\partial T}{\partial z} \right) + \dot{q} = \rho C_p \dfrac{\partial T}{\partial t}$

Spherical Coordinates:

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi}\left(k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta}\left(k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

Thermal Diffusivity: $\alpha = \dfrac{k}{\rho C_p}$

Thermal Resistance Concepts:

Conduction Resistance: $R_{t,cond} \overset{plane\,wall}{=} \dfrac{L}{kA}$; $R_{t,cond} \overset{cylinder}{=} \dfrac{\ln(r_o/r_i)}{2\pi l k}$; $R_{t,cond} \overset{sphere}{=} \dfrac{(1/r_i) - (1/r_o)}{4\pi k}$

Convection Resistance: $R_{t,conv} \overset{plane\,wall}{=} \dfrac{1}{h_{conv} A}$; $R_{t,conv} \overset{cylinder}{=} \dfrac{1}{2\pi r l h_{conv}}$; $R_{t,conv} \overset{sphere}{=} \dfrac{1}{4\pi r^2 h_{conv}}$

Radiation Resistance: $R_{t,rad} \overset{plane\,wall}{=} \dfrac{1}{h_{rad} A}$; $R_{t,rad} \overset{cylinder}{=} \dfrac{1}{2\pi r l h_{rad}}$; $R_{t,rad} \overset{sphere}{=} \dfrac{1}{4\pi r^2 h_{rad}}$

Combined Convection and Radiation Surface: $\dfrac{1}{R_{conv+rad}} = \dfrac{1}{R_{t,conv}} + \dfrac{1}{R_{t,rad}}$

Contact Resistance: $R_{t,contact} = \dfrac{1}{h_{contact} A_{contact}}$

Thermal Energy Generation: $T(x) - T_s \overset{plane\ wall}{=} \dfrac{\dot{q} L^2}{2k}\left(1 - \dfrac{x^2}{L^2} \right)$; $T(r) - T_s \overset{cylinder}{=} \dfrac{\dot{q} r_o^2}{4k}\left(1 - \dfrac{r^2}{r_o^2} \right)$

Extended Surfaces (fins with constant area of cross-section only):

$$\theta = T - T_\infty$$

Convective Tip: $\dfrac{\theta(x)}{\theta_b} = \dfrac{\cosh[m(L-x)] + (h/mk)\sinh[m(L-x)]}{\cosh(mL) + (h/mk)\sinh(mL)}$

$$q_{fin} = (hPkA_c)^{1/2}\,\theta_b\,\frac{\sinh(mL) + (h/mk)\cosh(mL)}{\cosh(mL) + (h/mk)\sinh(mL)}$$

Adiabatic Tip: $\dfrac{\theta(x)}{\theta_b} = \dfrac{\cosh[m(L-x)]}{\cosh(mL)}$; $q_{fin} = (hPkA_c)^{1/2}\,\theta_b\tanh(mL)$

Prescribed Tip Temperature: $\dfrac{\theta(x)}{\theta_b} = \dfrac{(\theta_L/\theta_b)\sinh(mx) + \sinh[m(L-x)]}{\sinh(mL)}$

$$q_{fin} = (hPkA_c)^{1/2}\,\theta_b\,\frac{\cosh(mL) - (\theta_L/\theta_b)}{\sinh(mL)}$$

Infinitely Long Fin: $\dfrac{\theta(x)}{\theta_b} = e^{-mx}$; $q_{fin} = (hPkA_c)^{1/2}\,\theta_b$

$$m^2 = \frac{hP}{kA_c};\ \theta_b = T_b - T_\infty\ ;\ q_{fin} = q_{conv,finsurface} + q_{conv,tip};\ q_{conv,tip} = hA_c\theta_L$$

Fin Effectiveness: $\varepsilon_{fin} = \dfrac{q_{fin}}{hA_{c,b}\theta_b}$; $\varepsilon_{fin} = \dfrac{R_{t,conv-base}}{R_{t,cond-fin}}$

Fin Efficiency: $\eta_{fin} = \dfrac{q_{fin}}{hA_{fin}\theta_b}$; $\eta_{fin} \overset{adiabatic}{=} \dfrac{\tanh(mL)}{mL}$; $L_c = L + \dfrac{A_c}{P}$; $\eta_{fin} = \dfrac{\tanh(mL_c)}{mL_c}$

$$\eta_o = \frac{q_{total}}{hA_{total}\theta_b} = 1 - \frac{NA_{fin}}{A_{total}}(1-\eta_{fin});\ R_{t,cond-fin} = \frac{1}{\eta_{fin}hA_{fin}};\ R_{t,cond-fin\,array} = \frac{1}{\eta_o hA_{total}}$$

Convection

Newton's Law of Cooling: $q''_{conv} = h_{conv}(T_s - T_\infty)$; $q_{conv} = q''_{conv}A$

Radiation

Emissive power = $E = \varepsilon\sigma T_s^4$

Irradiation received by surface from large surroundings: $G = \sigma T_{surr}^4$

Irradiation absorbed by surface = αG

Reflected irradiation: ρG

Gray surface: ε=α

Opaque surface: $\alpha + \rho = 1$

Radiative heat flux from a gray surface at T_S to a large surroundings at T_{surr}:

$$q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{surr}^4\right) = h_{rad}\left(T_s - T_{surr}\right)$$

$$h_{rad} = \varepsilon\sigma\left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right)$$

Useful Constants

σ = Stefan-Boltzmann's Constant = $5.67\times10^{-8}\ \frac{\mathrm{W}}{\mathrm{m^2K^4}}$

Geometry

Cylinder: $A = 2\pi rl$; $V = \pi r^2 l$

Sphere: $A = 4\pi r^2$; $V = \frac{4}{3}\pi r^3$