Name: ________________________________

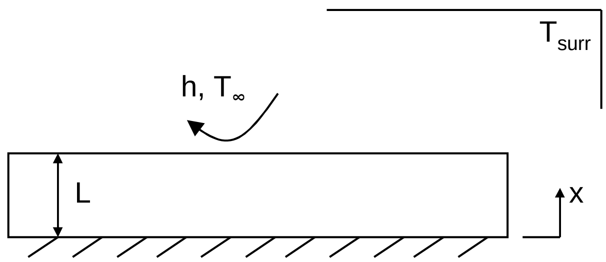
ME 315: Heat and Mass Transfer
Spring 2008
EXAM 1
Wednesday, 13 February 2008
7:00 to 8:00 PM

Instructions: This is an open-book exam. You may refer to your course textbook, your class notes and your graded homework assignments. Show all work clearly for maximum credit. **CIRCLE ALL ANSWERS**.

Problem	Score
1	/ 20
2	/ 20
3	/ 20
Total	/ 60

Name: ________________________________

1. (20 points) During an annealing process, a defect has been detected in a steel slab (approximated as a plane wall) of thickness L. Once the slab exits the furnace, it is to be set aside to cool before being reworked. When the slab exits the furnace, it is at a uniform temperature of T_i = 1000°C and the air and surroundings are at a temperature of $T_\infty = T_{surr}$ = 30°C providing a convection coefficient of 40 W/m^{2}K. The hot slab is placed such that the lower surface of the slab may be considered insulated.

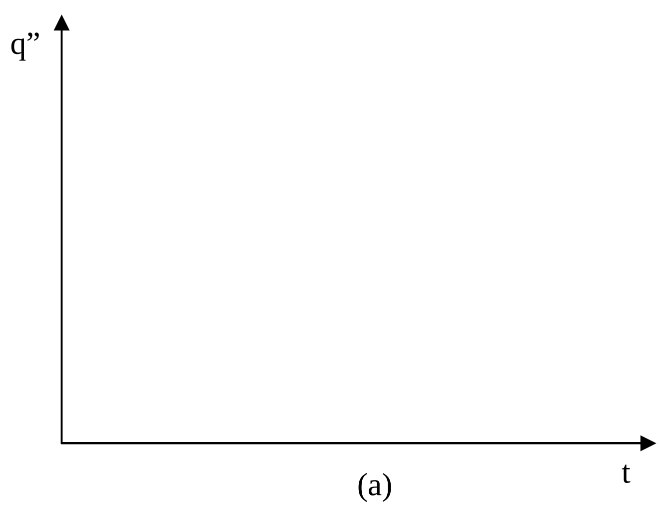


   a. (4 pts.) Sketch the heat flux as a function of time, q"(t), at the top surface of the slab, x = L, using the axes given below. Label and describe important features.

   b. (6 pts.) Assuming the slab behaves as a blackbody, what is the initial heat flux at x = L and t = 0?

   c. (4 pts.) If the wall has a thermal conductivity of 50 W/m·K, what is the corresponding temperature gradient at x = L, and t = 0?

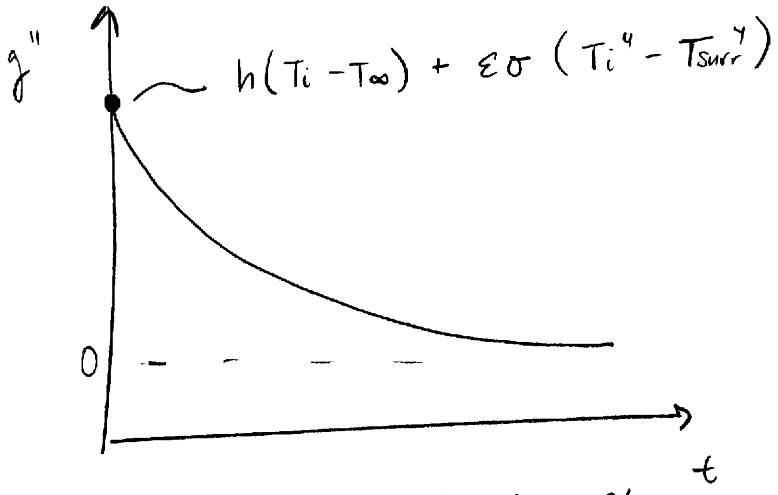
   d. (6 pts.) For a plate thickness of L = 20 cm (ρ = 7800 kg/m^3, c_p = 500 J/kg·K), calculate the amount of energy per unit plate surface area (J/m^2) that has been lost to the surroundings by the time steady-state conditions have been reached.



(a)

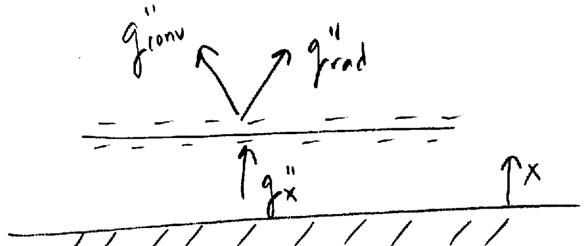
(1.)

a.)



The heat flux at $x = L$ starts at a maximum and decreases with time and slows as the temperature approaches $T_\infty = T_{surr}$.

b.)



Control surface at $x = L$

$$\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_g}^{\,0} = \cancel{\dot{E}_{st}}^{\,0}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$T_i = 1000\,°C = 1273\text{ K}$

$T_\infty = T_{surr} = 30\,°C = 303\text{ K}$

$$q''_x = q''_{rad} + q''_{conv}$$

$$q''_x = h(T_i - T_\infty) + \varepsilon\sigma(T_i^4 - T_{surr}^4)$$

$$q''_x = (40\ \mathrm{W/m^2 \cdot \cancel{K}})(1000 - 30)\cancel{°C} + (1)(5.67\times10^{-8}\ \frac{\mathrm{W}}{\mathrm{m^2 \cdot \cancel{K^4}}})(1273^4 - 303^4)\cancel{K^4}$$

$$q''_x = 187.2\ \frac{\mathrm{kW}}{\mathrm{m^2}}$$

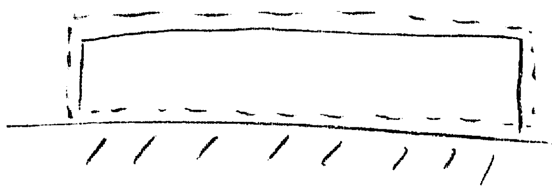
c.) $$q_x'' = -k\frac{dT}{dx} = h(T_i - T_\infty) + \varepsilon\sigma\left(T_i^4 - T_{surr}^4\right)$$

$$\frac{dT}{dx} = \frac{h(T_i - T_\infty) + \varepsilon\sigma\left(T_i^4 - T_{surr}^4\right)}{-k}$$

$$\frac{dT}{dx} = \frac{187.2 \times 10^3 \ W/m^2}{-50 \ W/m\cdot K}$$

$$\frac{dT}{dx} = -3744 \ \frac{K}{m}$$

d.) for a control volume over the whole slab:



$$\cancel{\dot{E}_{in}}^{\,0} - \dot{E}_{out} + \cancel{\dot{E}_g}^{\,0} = \dot{E}_{st}$$

$$\dot{E}_{out} = -\dot{E}_{st} = -\rho c \forall \frac{dT}{dt}$$

$$\int_t^0 \dot{E}_{out}\, dt = -\rho c A L \int_{T_i}^{T_f} dT$$

$$\dot{E}_{out}\, dt = E_{out} = \rho c A L (T_f - T_i)$$

$$\frac{E_{out}}{A} = E_{out}'' = \rho c L (T_f - T_i)$$

$$E_{out}'' = -\left(7800 \ \frac{kg}{m^3}\right)\left(500 \ J/kg\cdot K\right)(0.2 \ m)(30 - 1000)\,°C$$

$$E_{out}'' = 7.57 \times 10^8 \ \frac{J}{m^2}$$

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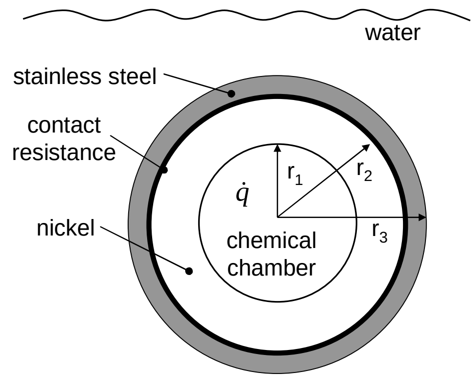
2. (20 points) It is proposed that the temperature for a chemical reaction be controlled by placing the reactants in a long, cylindrical container that is submerged in water (h = 200 W/m$^2\cdot$K, T_{water} = 20°C). The chemical chamber is made of a nickel wall and is encased in an AISI 302 stainless steel shell. There is contact resistance present between the nickel and stainless steel shells of $R_c' = 0.003\ \mathrm{K \cdot m/W}$ (per unit length of the cylinder). Heat transfer out of the ends of the cylindrical container may be neglected.



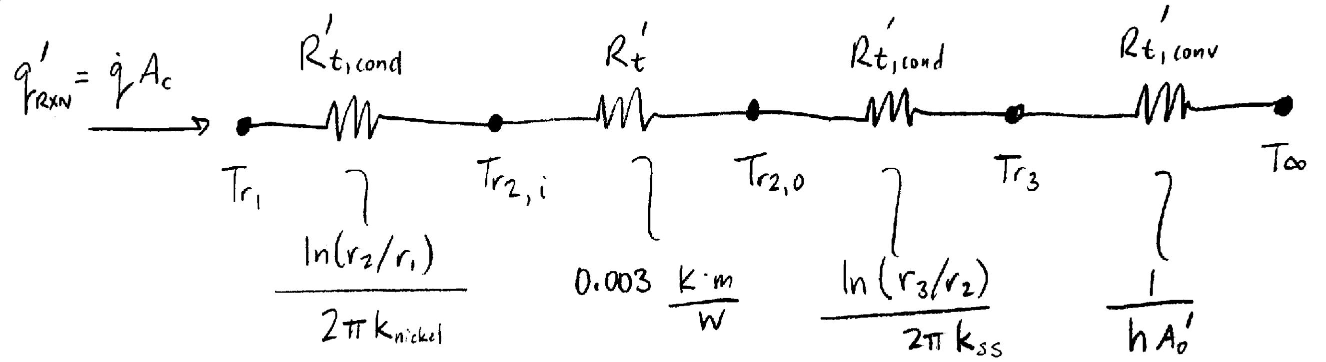
$r_1 = 5$ cm, $r_2 = 6$ cm, $r_3 = 6.5$ cm

a. (4 pts.) Sketch the thermal circuit for the heat loss from the chemical chamber to the environment and label important features.

b. (8 pts.) Find the temperature at the inner surface of the nickel cylinder ($r = r_1$) for $\dot{q} = 1\text{x}10^6\ \mathrm{W/m^3}$.

c. (8 pts.) Find the maximum temperature in the chemical mixture if its thermal conductivity is k = 10 W/m$\cdot$K. Assume there is no convection in the mixture itself, and heat transfer through the mixture is only by conduction.

(2.) a.)



b) Performing a surface energy balance at $r = r_1$

$$\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_{gen}}^{\,0} = \cancel{\dot{E}_{ST}}^{\,0}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$q'_{RXN} = \frac{T_{r_1} - T_\infty}{\frac{\ln(r_2/r_1)}{2\pi k_{nickel}} + R'_c + \frac{\ln(r_3/r_2)}{2\pi k_{ss}} + \frac{1}{hA'_o}}$$

where $q'_{RXN} = \dot{q}(\pi r_1^2)$, $k_{ss} = 15.1\ W/m \cdot K$

and $A'_o = 2\pi r_3$, $k_{nickel} = 90.7\ W/m \cdot K$

$$\therefore\ T_{r_1} = \dot{q}(\pi r_1^2)\left[\frac{\ln(r_2/r_1)}{2\pi k_{nickel}} + R'_c + \frac{\ln(r_3/r_2)}{2\pi k_{ss}} + \frac{1}{h(2\pi r_3)}\right] + T_\infty$$

substituting values:

$$T_{r_1} = \left(1\times10^6 \frac{W}{m^3}\right)\pi(0.05m)^2\left[\frac{\ln(6/5)}{2\pi(90.7 \frac{W}{mK})} + 0.003\ \frac{K \cdot m}{W}\right.$$

$$\left. + \frac{\ln(6.5/6)}{2\pi(15.1\ W/m \cdot K)} + \frac{1}{(200 \frac{W}{m^2 \cdot K})(2\pi)(0.065m)}\right] + 20^\circ C$$

$T_{r_1} = 149\,^\circ C = 422\ K$

c.) for a cylinder (control volume of $r=0 \longrightarrow r=r_1$)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

negligible changes in ϕ and z

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \dot{q} = 0$$

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$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{-\dot{q}r}{k}$$

$$r\frac{\partial T}{\partial r} = \frac{-\dot{q}r^2}{2k} + C_1$$

$$\frac{\partial T}{\partial r} = \frac{-\dot{q}r}{2k} + \frac{C_1}{r}$$

$$T(r) = \frac{-\dot{q}r^2}{4k} + C_1 \ln(r) + C_2 \qquad \boxed{\text{eq. } 3.51}$$

students may start here.

B.C. $\begin{cases} \left.\frac{dT}{dr}\right|_{r=0} = 0 \longrightarrow C_1 = 0 \\ T(r=r_1) = T_{r_1} \end{cases}$

$$T_{r_1} = \frac{-\dot{q}r_1^2}{4k} + C_2$$

$$C_2 = T_{r_1} + \frac{\dot{q}r_1^2}{4k}$$

$$\therefore \quad T(r) = \frac{-\dot{q}r^2}{4k} + \frac{\dot{q}r_1^2}{4k} + T_{r_1}$$

$$= \frac{\dot{q}}{4k}\left(r_1^2 - r^2\right) + T_{r_1}$$

$$T(r=0) = \frac{\dot{q}r_1^2}{4k} + T_{r_1}$$

$$= \frac{(1\times10^6\ \text{W/m}^3)(0.05\ \text{m})^2}{4(10\ \text{W/m}\cdot\text{K})} + 149^\circ\text{C}$$

$$T(r=0) = 211.5^\circ\text{C} = 484.5\,\text{K}$$

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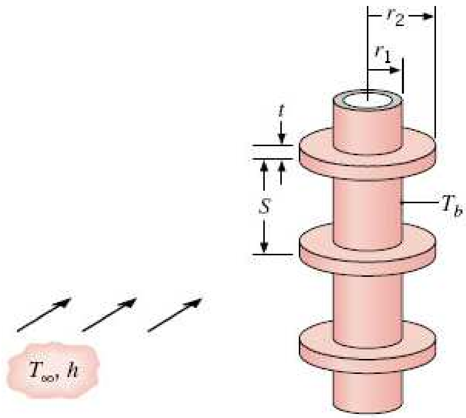
3. (20 points) The addition of annular fins (with rectangular profile) to the exterior of a pipe is being considered to cool hot gases running through the pipe core. The fins are t = 2.5 mm thick and L = 10 mm long ($L = r_2 - r_1$). For a total tube length of L_t = 0.5 m and a fin pitch of S = 10 mm, there are N = 50 fins. The outer tube radius without fins is r_1 = 10 mm. The fins and tube are made of aluminum, k = 240 W/m·K.



a. (14 pts.) What is the percentage increase in heat transfer with the fins attached compared to the case with no fins on the tube? Assume that the convection coefficient with the fins attached is h_w = 50 W/m^2·K and without the fins $h_{w/o}$ = 75 W/m^2·K.

b. (6 pts.) What is the fin effectiveness and rate of heat transfer (with fins attached) for the tube length of 0.5 m if we assume that T_b = 100°C and T_∞ = 20°C. You may neglect radiation.

(3.) a.)

$L = 10$ mm

$t = 2.5$ mm

$r_1 = 10$ mm

$h_w = 50$ W/m²·K

$h_{w,o} = 75$ W/m²·K

$k = 240$ W/m·K

$N = \frac{L_t}{S} = \frac{0.5\text{ m}}{0.01\text{ m}} = 50$

$r_2 = L + r_1 = 20$ mm

$r_2/r_1 = 20/10 = 2$

$L_c = L + t/2 = 10 + \frac{2.5}{2} = 11.25$ mm

$r_{2,c} = r_2 + t/2 = 20 + 2.5/2 = 21.25$ mm

$A_p = L_c t = (11.25\text{ mm})(2.5\text{ mm}) = 28.125\text{ mm}^2$ (see pg. 149)

$$(L_c)^{3/2}\left(\frac{h_w}{kA_p}\right)^{1/2} = (.01125)^{3/2}\left[\frac{50}{240\,(2.813\times10^{-5})}\right]^{1/2} = 0.103$$

from figure 3.19 with $r_2/r_1 = 2$

$\longrightarrow \eta_f \approx 0.96$

$$q_f = \eta_f\, q_{max} = \eta_f\, h\, A_f\, \theta_b \qquad (3.86)$$

$$q_f = \eta_f\, h\left[2\cdot\pi\left(r_{2,c}^2 - r_1^2\right)\right]\theta_b$$

$$q_f = (0.96)(50\text{ W}/\cancel{\text{m}^2}\cdot\text{K})\left[2\pi\left(.02125^2 - 0.01^2\right)\cancel{\text{m}^2}\right]\theta_b$$

$$q_f = (0.106\text{ W/K})\,\theta_b$$

total q_w with fins:

$$q_w = Nq_f + A_b h_w \theta_b = Nq_f + \left[(L_t - Nt)\,2\pi r_1\right] h_w \theta_b$$

$$q_w = 50\left(0.106\ \mathrm{W/K}\right)\theta_b + \left[\left(0.5\,\mathrm{m} - 50\left(0.0025\,\mathrm{m}\right)\right)\right] 2\pi\left(0.01\,\mathrm{m}\right)\left(50\ \mathrm{W/m^2K}\right)\theta_b$$

$$q_w = \left(5.30\ \mathrm{W/K}\right)\theta_b + \left(1.18\ \mathrm{W/K}\right)\theta_b$$

$$q_w = \left(6.48\ \mathrm{W/K}\right)\theta_b$$

total $q_{w,o}$ without fins :

$$q_{w,o} = h_{w,o} A\,\theta_b = h_{w,o}\left[\left(2\pi r_1\right)L_t\right]\theta_b$$

$$= 75\ \mathrm{W/m^2\cdot K}\left[2\pi\left(0.01\,\mathrm{m}\right)\left(0.5\,\mathrm{m}\right)\right]\theta_b$$

$$= \left(2.36\ \mathrm{W/K}\right)\theta_b$$

percent increase :

$$\frac{q_w - q_{w,o}}{q_{w,o}} = \frac{\left(6.48 - 2.36\right)\left(\cancel{\mathrm{W/K}}\right)\cancel{\theta_b}}{\left(2.36\ \cancel{\mathrm{W/K}}\right)\cancel{\theta_b}} = 1.74 \text{ or } 174\% \text{ increase}$$

b.)

$$\varepsilon_f = \frac{q_f}{h_w A_{c,b}\,\theta_b} \qquad (3.81)$$

$$\varepsilon_f = \frac{\left(0.106\ \mathrm{W/K}\right)\cancel{\theta_b}}{\left(50\ \mathrm{W/\cancel{m^2}\cdot K}\right)\left(2\pi\right)\left(0.01\,\cancel{m}\right)\left(0.0025\,\cancel{m}\right)\cancel{\theta_b}} = 13.5$$

$$q_w = N q_f + A_b h_w \theta_b = N q_f + \left[\left(L_t - Nt\right)2\pi r_1\right] h_w \theta_b$$

$$q_w = \left(6.48\ \mathrm{W/K}\right)\left(100\,^\circ\mathrm{C} - 20\,^\circ\mathrm{C}\right) = 518.4\ \mathrm{W}$$