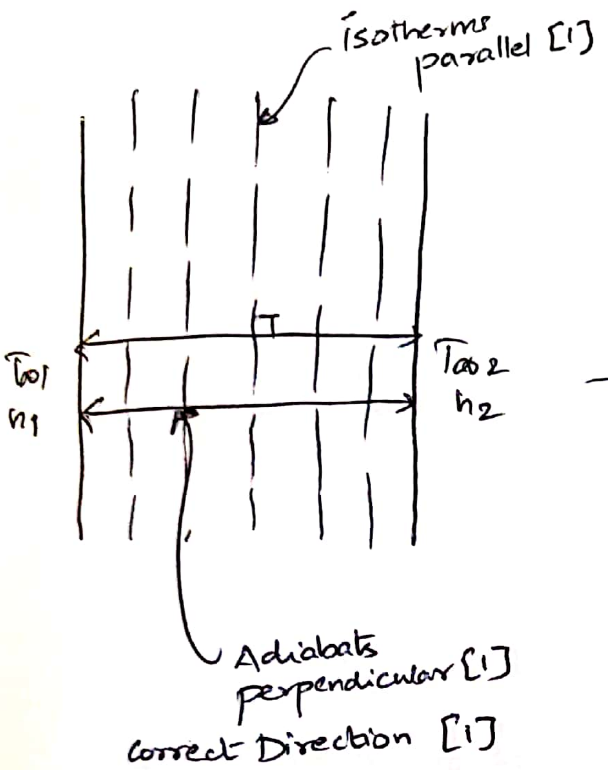
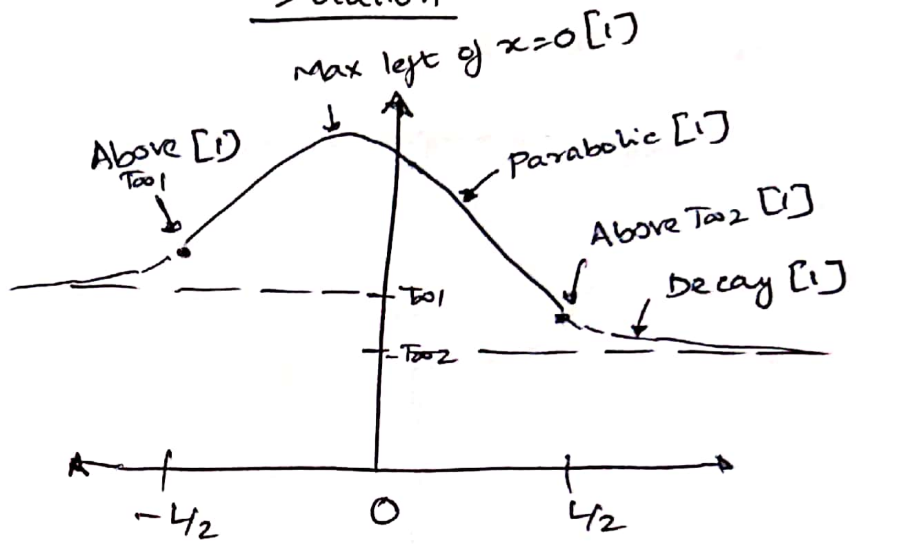
ME 315 – EXAM I

Q1 (Heat generation in a slab)



$\left(T_{(x=-\frac{L}{2})} > T_{(x=L/2)}\right)$ [1]

Solution

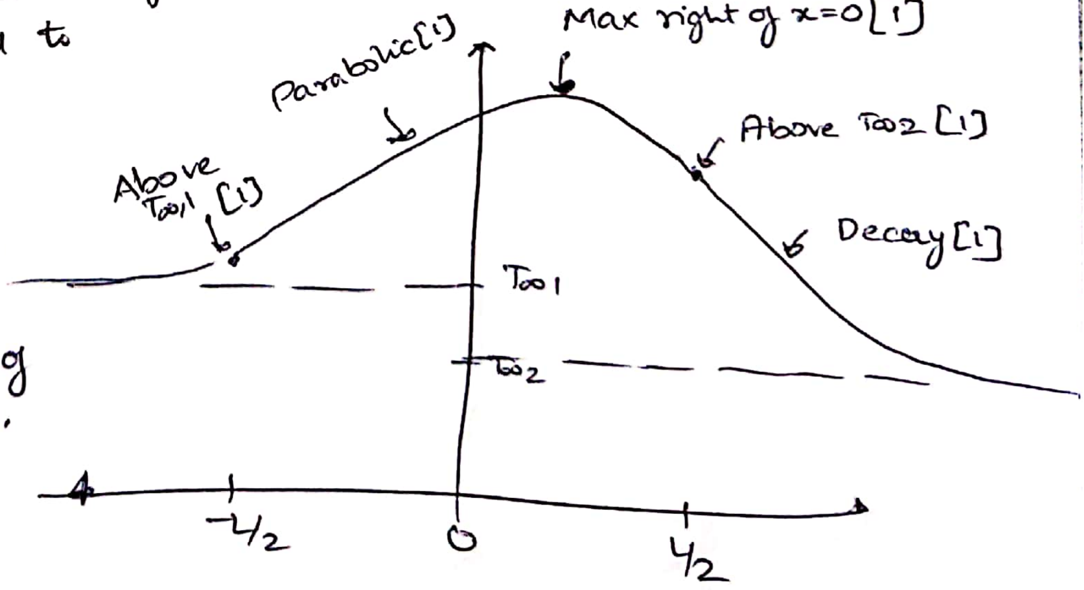


Notes

1. The alternate solution is possible if convection resistance on the right is very large compared to conduction resistance (very small h_2)
2. In all cases, the solution is parabolic, so the Max. temperature and side of the peak must match.

Alternative Solution

$\left(T_{(x=L/2)} > T_{(x=-L/2)}\right)$ [1]



Justification

For 1D steady conditions, the diffusion equation becomes,

$$k \frac{\partial^2 T}{\partial x^2} + \dot{q} = 0 \quad \leftarrow [1]$$

Solution $T(x) = -\frac{\dot{q}}{2k} x^2 + C_1 x + C_2$ ∴ parabolic profile [1]

Since boundary conditions are asymmetric, the profile will be asymmetric parabolic. Possible solutions are : (Max temperature shifted to left, or right of $x=0$).

↑
Justify asymmetry [1].

(or)

Justify the above equations in words
+ [3].
Justify asymmetry

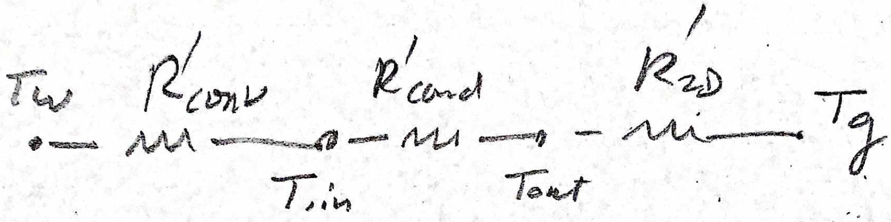
Prob. 2

Assumptions:

- steady state
- radiation is negligible

2.a

4 pts



2 pts $\quad R'_{conv} = \dfrac{1}{h \cdot 2\pi r_o \cdot 1} = \dfrac{1}{40 \cdot 2\pi \cdot 0.02} = 0.199 \ m \cdot K/W$

2 pts $\quad R'_{cond} = \dfrac{\ln r_o/r_i}{2\pi k_p \cdot 1} = \dfrac{\ln 2.5/2}{2\pi \cdot 50} = 0.0007 \ m \cdot K/W$

2 pts $\quad R'_{2D} = \dfrac{1}{S k_g} = \dfrac{\ln 4z/D}{2\pi \cdot 1 \cdot k} = \dfrac{\ln \frac{4 \times 0.2}{0.05}}{2\pi \cdot 2} = 0.2206 \ m \cdot K/W$

2.b: $\quad q' = \dfrac{T_w - T_g}{\Sigma R'} = \dfrac{40}{0.4203} = 95.17 \ W/m$

8 pts $\qquad$ 2 pts

2.c $\quad q' = h \cdot 2\pi r_i (T_w - T_{in})$ $\qquad$ 8 pts

$$T_{in} = T_w - \frac{q'}{h \, 2\pi r_i} = 20 - \frac{95.17}{40 \cdot 2 \cdot \pi \cdot 0.02}$$

$$= 1.07 \ ^\circ C$$ 2 pts

There are different ways to do this.

(I) Assumptions :

Uniform heat generation, 1D, Steady state in $(0, L_1)$ +1

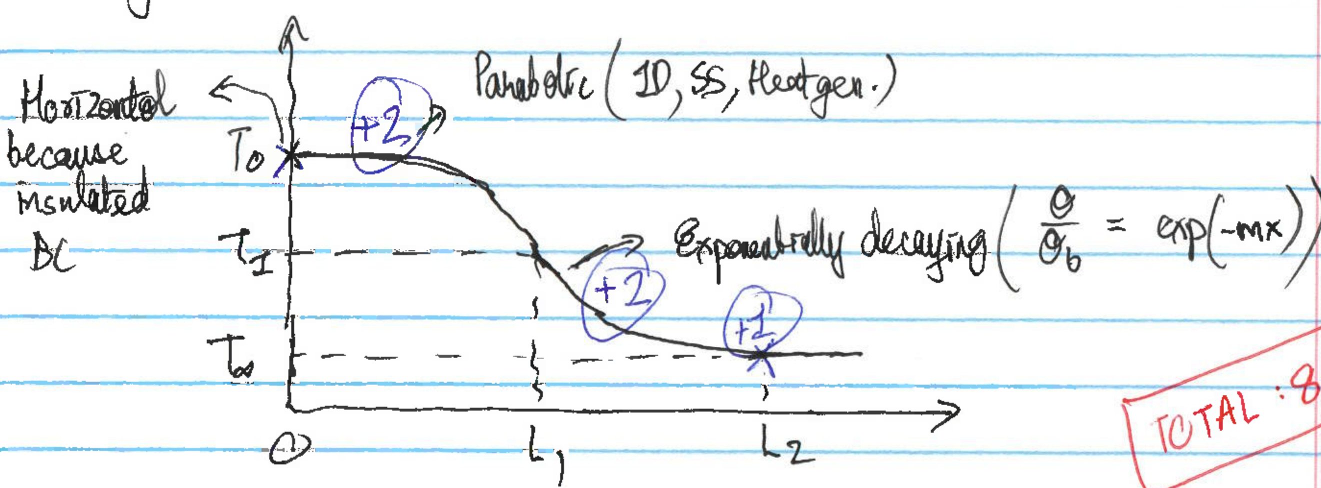
Infinitely long fin from $L_1 < x < L_2$

(a) Using symmetry, $x = 0$ will be insulated.

Parabolic downward from $x = 0$ to $x = L_1$, with zero gradient at $x = 0$.

Exponentially decaying from $x = L_1$ to $x = L_2$ due to long fin to ambient temperature

Temperature profile becomes horizontal at and after $x = L_2$ because long fin



TOTAL : 8

Max temperature at $x = 0$ +1

Minimum temperature at $x = L_2$ +1

(b)

+2 $\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \Rightarrow \frac{dT}{dx} = -\frac{\dot{q}}{k}x + C_1$

$\Rightarrow T = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2$ +2

$x = 0$, $dT/dx = 0 \Rightarrow C_1 = 0$ +1

$x = L_1,\ T = T_1 \Rightarrow T_1 = -\frac{\dot{q}}{2k}L_1^2 + C_2 \Rightarrow C_2 = T_1 + \frac{\dot{q}}{2k}L_1^2$ +1

$$\therefore T(x) = -\frac{\dot{q}}{2k}x^2 + T_1 + \frac{\dot{q}}{2k}L_1^2$$

TOTAL : 8

$$\therefore T(x=0) = T_0 = T_1 + \frac{\dot{q}}{2k}L_1^2$$ +2

(c) Portion of the rod beyond the coil $L_1 \le x \le L_2$ behaves as an +2 infinitely long fin for which heat rate is

$$q_f = q_x(L_1) = (hPkA_c)^{1/2}(T_1 - T_\infty)$$ +2

$\because$ Cylindrical fin $\Rightarrow P = \pi D,\ A_c = \frac{\pi D^2}{4}$ +1

Here, $h = 20\ W/m^2\text{-}K,\ k = 50\ W/m\text{-}K,\ D = 10mm = 0.01\ m$

Energy balance on embedded portion of the rod

TOTAL : 8

$$\cancel{q_{in}} - q_{out} + q_{gen} = 0$$

$$\Rightarrow q_{gen} = q_{out} \quad (q_{out} = q_{fin})$$ +1

$$\Rightarrow \dot{q}A_cL_1 = (hPkA_c)^{1/2}(T_1 - T_\infty)$$

$$\Rightarrow T_1 = T_\infty + \frac{\dot{q}A_cL_1}{(hPkA_c)^{1/2}}$$ +2

(d) Substituting numerical values, $T_1 = 222.6424\ °C$ +3

$T_c = 253.8924$ +3

TOTAL : 6