Write Down Your NAME

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Last First

Circle Your DIVISION

Div. 1	**Div. 2**	**Div. 3**	**Div.4**
8:30 am	**9:30 pm**	**12:30 pm**	**3:30 pm**
Han	**Xu**	**Ruan**	**Pan**

ME315 Heat and Mass Transfer
School of Mechanical Engineering
Purdue University

Exam 1
September 28, 2017

Read Instructions Carefully:

- Write your name on **each page** and circle your division number.
- Equation sheet and tables are attached to this exam. One page of letter-size crib sheet is allowed.
- No books, notes, and other materials are allowed.
- ME Exam Calculator Policy is enforced. Only TI-30XIIS and TI-30XA are allowed.
- **Power off** all other digital devices, such as computer/tablet/phone and smart watch/glasses.
- Keep all the pages in order.
- You are asked to write your assumptions and answers to sub-problems in designated areas. Write on front side of the page only. If needed, you can insert extra pages but mark this clearly in the designated areas.

Performance		
1	**35**	
2	**30**	
3	**35**	
Total	**100**	

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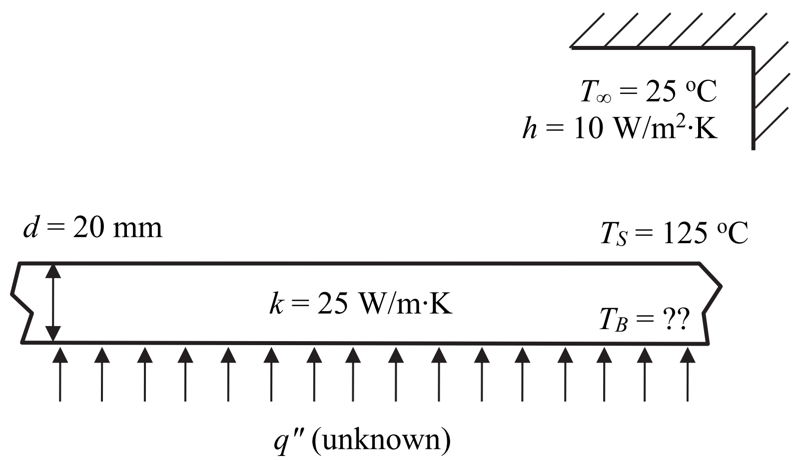
Problem 1 [35 points]

At steady state, a large steel plate is placed in ambient air with a temperature T_∞ = 25 °C. The steel plate has a thermal conductivity k = 25 W/m·K and a thickness d = 20 mm. The bottom surface of the plate is covered by an electrical heater supplying a uniform heat flux of q'' without convection or radiation heat losses. The top surface of the plate has a temperature T_s = 125 °C, and is subjected to convection with a coefficient of h = 10 W/m^2·K. The top surface is polished therefore the radiation heat exchange is negligible.



(a) Draw a thermal circuit of this 1-D problem with symbolical labels.

(b) Calculate the values of the individual thermal resistances in part (a).

(c) Find the temperature T_B at the bottom of the plate.

List your assumptions below. [3 pts]

Assumptions:

Steady state, no internal heat generation, 1D conduction, negligible radiation, etc.

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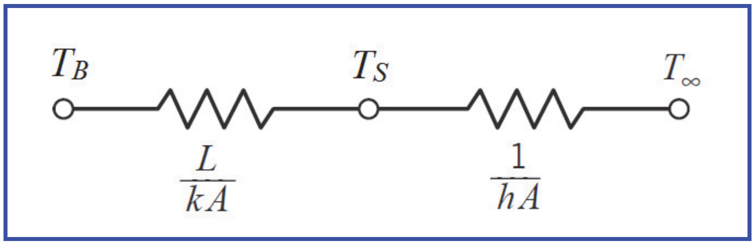
Problem 1 – continued

Start you answer to part (a) here. [12 pts]



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Problem 1 – continued

Start your answer to part (b) here. [10 pts]

Assume unit area, i.e. A=1m^2

$$R_{t,cond} = \frac{L}{kA} = \frac{0.02}{25\times 1} = 0.0008\left[\frac{K\cdot m}{W}\right]$$

$$R_{t,conv} = \frac{1}{hA} = \frac{1}{10\times 1} = 0.1\left[\frac{K\cdot m}{W}\right]$$

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Problem 1 – continued

Start your answer to part (c) here. [10 pts]

Assume unit area, i.e. A=1m^2

$$q = \frac{T_S - T_\infty}{R_{Total}} = \frac{125-25}{0.1} = 1,000[W]$$

$$T_B = T_S + qR_{t,cond} = 125 + 1,000 \times 0.0008 = 125.8\left[{}^oC\right]$$

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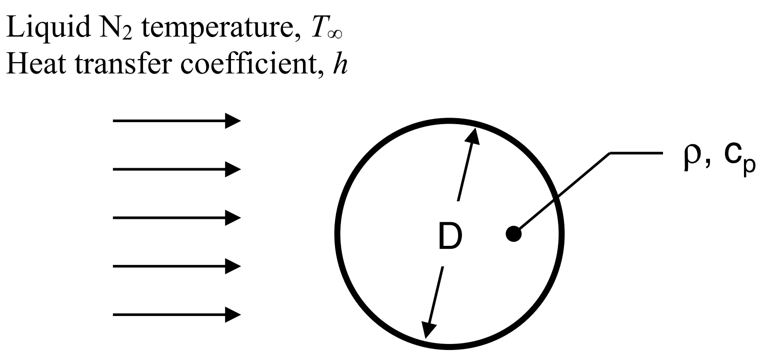
Problem 2 [30 points]

A copper sphere of diameter D, density ρ, and specific heat c_p is initially at a temperature T_i when it is suddenly submerged in liquid nitrogen at its boiling temperature T_∞. Unlike most convective cooling situations where the convective heat transfer coefficient h is fairly independent of temperature, boiling is associated with a heat transfer coefficient that varies with the surface and fluid temperatures as follows:

$$h = G\left(T_s - T_\infty\right)^2 \text{ and } q_{conv} = hA_S\left(T_s - T_\infty\right)$$

where G is a known constant. Assuming the sphere behaves as a lumped mass (i.e., Bi << 0.1), answer the following questions.



(a) Derive a differential equation that will give the temperature of the copper sphere, $T(t)$, as a function of time, t, by applying the energy balance analysis around the copper sphere.

(b) Determine the temperature $T(t)$ by solving the differential equation obtained from Part (a).

List your assumptions below. [5 pts]

Assumptions:

Uniform temperature inside the sphere (lumped temperature), transient response, negligible radiation, constant properties, etc.

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Problem 2 – continued

Start you answer to part (a) here. [15 pts]

Balance energy (control volume) over the whole sphere:

$$\cancel{\dot{E}_{in}} + \cancel{\dot{E}_{gen}} - \dot{E}_{out} = \dot{E}_{st}$$

$$-h\left(T - T_\infty\right) = \rho c V \frac{dT}{dt}$$

$$-GA\left(T - T_\infty\right)^3 = \rho c V \frac{dT}{dt}$$

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Problem 2 – continued

Start your answer to part (b) here. [10 pts]

$$-GA(T-T_\infty)^3 = \rho c V\frac{dT}{dt}$$

$$\Rightarrow -GA(T-T_\infty)^3 = \rho c V\frac{dT}{dt}$$

$$\Rightarrow \frac{-GA}{\rho c V}dt = \frac{1}{(T-T_\infty)^3}dT$$

$$\Rightarrow \int\frac{-GA}{\rho c V}dt = \int\frac{1}{(T-T_\infty)^3}dT$$

$$\Rightarrow \frac{-GA}{\rho c V}t = \frac{-1}{2(T-T_\infty)^2} + C_1$$

Initial condition: $T(0)=T_i$

$$\frac{-1}{2(T_i-T_\infty)^2} + C_1 = 0$$

$$\Rightarrow C_1 = \frac{1}{2(T_i-T_\infty)^2}$$

$$\Rightarrow \frac{-GA}{\rho c V}t = \frac{1}{2(T_i-T_\infty)^2} - \frac{1}{2(T-T_\infty)^2}$$

$$\Rightarrow T(t) = \left(\frac{1}{(T_i-T_\infty)^2} + \frac{2GA}{\rho c V}t\right)^{-0.5} + T_\infty$$

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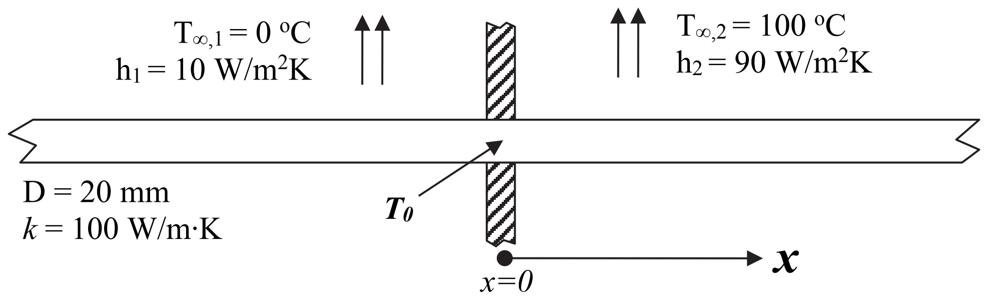
Problem 3 [35 points]

As shown in the figure below, a very long rod of 20-mm diameter and uniform thermal conductivity k = 100 W/m·K is placed across two gas chambers separated by a ***thin*** insulation wall. The parameters are shown in the figure below. The rod can be treated as two infinite fins ($x > 0$ and $x < 0$) of same material connected at their bases. Assume steady state and neglect radiation.



(a) Qualitatively sketch the temperature variation of the rod, where $-\infty < x < +\infty$. Label all important features in the temperature profile.

(b) Find the expressions of fin heat rates to each sides $q_{f,1}$ and $q_{f,2}$ in terms of T_0 and the parameters provided (T_0, $T_{\infty,1}$, $T_{\infty,2}$, h_1, h_2, D, and k), assuming both sides of the rod can be approximated as infinite fins.

(c) Calculate the value of the temperature T_0 of the rod at the location of the wall (x = 0).

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.70)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.72)
B	Adiabatic $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$ (3.75)	$M \tanh mL$ (3.76)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.77)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.78)
D	Infinite fin ($L \to \infty$): $\theta(L) = 0$	e^{-mx} (3.79)	M (3.80)

$\theta \equiv T - T_\infty$ $m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $M \equiv \sqrt{hPkA_c}\theta_b$

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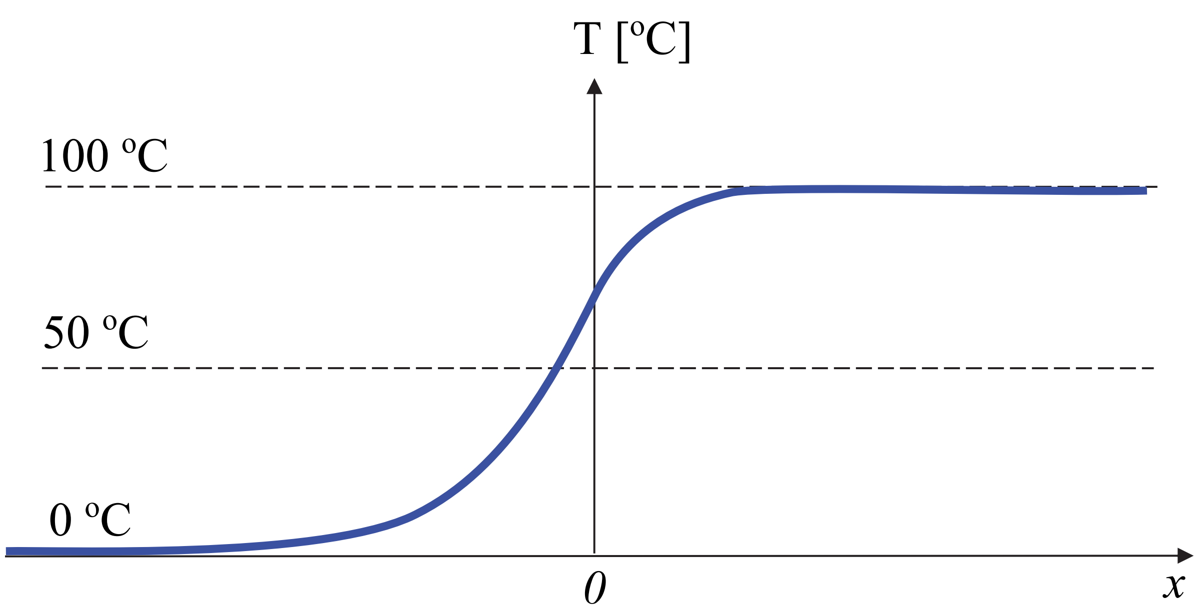
Problem 3 – continued

List your assumptions below. [3 pts]

Assumptions:

1D conduction, neglect radiation, infinite fin approximation, no heat conduct from the rod to the wall, constant properties, etc.

Start you answer to part (a) here. [12 pts]



Key features of the temperature profile:

- Exponentially approaches to 0°C at left hand side;
- Exponentially approaches to 100°C at right hand side;
- The decay rate for the right hand side is faster than that of the left hand side;
- Temperature continuous at *x*=0.
- Slope of temperature continuous (temperature profile is smooth) at *x*=0.
- Temperature value is higher than 50°C at x=0.

Problem 3 – continued

Start your answer to part (b) here. [10 pts]

Left hand side (x<0)

$$q_{f,1} = M_1 = \sqrt{h_1 P k A_c}\,\theta_{b,1} = \sqrt{h_1 \pi D k \left(\pi D^2/4\right)} \cdot \left(T_0 - T_{\infty,1}\right)$$

$$= \sqrt{h_1 \pi^2 D^3 k/4} \cdot \left(T_0 - T_{\infty,1}\right)$$

Right hand side (x>0)

$$q_{f,2} = M_2 = \sqrt{h_2 P k A_c}\,\theta_{b,2} = \sqrt{h_2 \pi D k \left(\pi D^2/4\right)} \cdot \left(T_{\infty,2} - T_0\right)$$

$$= \sqrt{h_2 \pi^2 D^3 k/4} \cdot \left(T_{\infty,2} - T_0\right)$$

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Problem 3 – continued

Start your answer to part (c) here. [10 pts]

Do energy balance at the junction of two fins (x=0)

$$q_{f,1} = q_{f,2}$$

$$\Rightarrow \sqrt{h_1 P k A_c}\left(T_0 - T_{\infty,1}\right) = \sqrt{h_2 P k A_c}\,\theta_{b,2}\left(T_{\infty,2} - T_0\right)$$

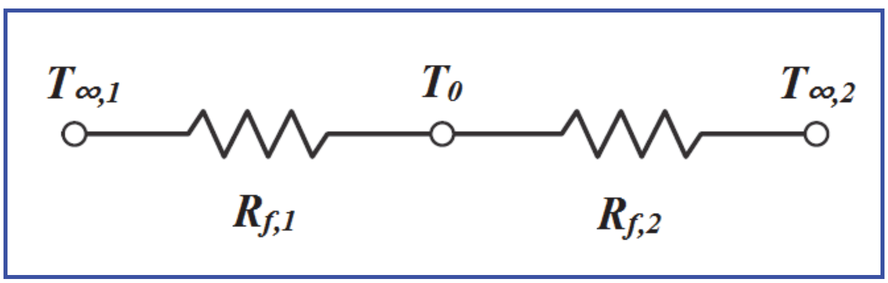
$$\Rightarrow \sqrt{h_1}\left(T_0 - T_{\infty,1}\right) = \sqrt{h_2}\left(T_{\infty,2} - T_0\right)$$

$$\Rightarrow \left(T_0 - 0\right) = 3\left(100 - T_0\right)$$

$$\Rightarrow T_0 = 75 \quad \left[{}^{O}C\right]$$

Note: The signs of $q_{f,1}$ and $q_{f,2}$ can be different.

Alternatively, one can use fin resistance analysis:



Where,

$$R_{f,1} \triangleq \frac{T_0 - T_{\infty,1}}{q_{f,1}}; \quad R_{f,2} \triangleq \frac{T_{\infty,2} - T_0}{q_{f,2}}$$

From here, one can find the T_0 based on circuit analysis.