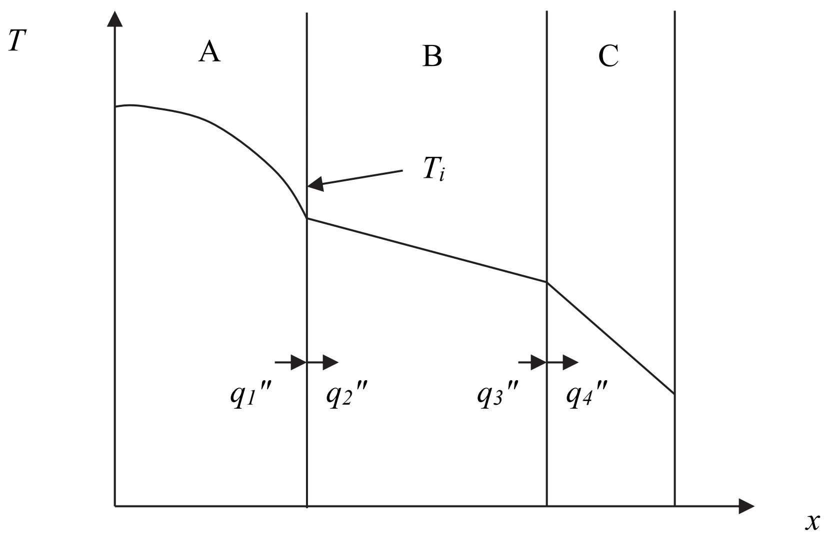
Problem 1 [35 points]

The steady-state temperature distribution in a 1-D composite plane wall of three different materials, each of constant but different thermal conductivity, k_A, k_B, and k_C, is shown in the figure below. Material A has a uniform volumetric heat generation $\dot{q}$ [W/m^3]. The thicknesses of the walls in A, B, and C are L_A, L_B, and L_C, respectively. The left side of the wall is insulated, and the right side of the wall is subjected to convection cooling, with T_∞ and h.

(a) Comment on the relative magnitudes of q_1'' and q_2'', and relative magnitudes of q_3'' and q_4''. Provide a brief justification.
(b) Comment of the relative magnitudes of k_A and k_B, and of k_B and k_C, and provide a brief justification.
(c) Sketch heat flux q'' as a function of x, in the figure on the next page.
(d) Find the expression of the temperature at the interface between wall A and wall B, T_i, in terms of given parameters $\dot{q}$, L_A, L_B, L_C, k_A, k_B, k_C, h, and T_∞.



List your assumptions below. [3 pts]

- Radiation Negligible
- Constant resistances or material properties
- No contact thermal resistance

Start you answer to part (a) here. [6 pts]

$$q_1'' = q_2''$$
$$q_3'' = q_4''$$

Due to energy balance or energy conservation at the interfaces

Problem 1 – continued

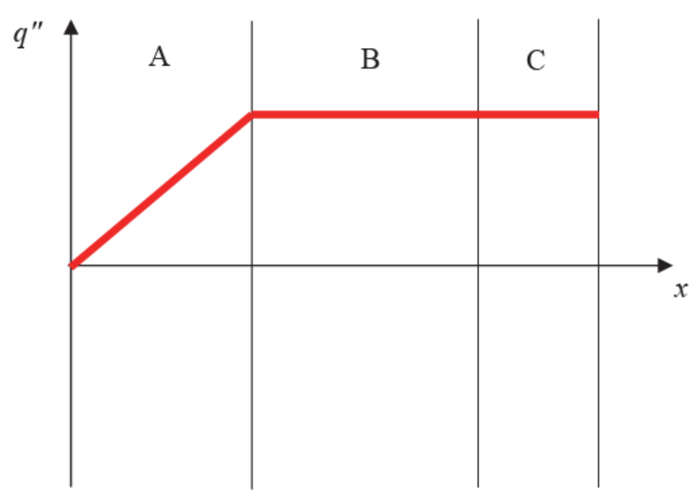
Start your answer to part (b) here. [6 pts]

From (a), $q_1'' = q_2'' \quad \Rightarrow \quad -k_A \left.\frac{dT}{dx}\right|_A = -k_B \left.\frac{dT}{dx}\right|_B$

Also, from the figure $\left|\frac{dT}{dx}\right|_A > \left|\frac{dT}{dx}\right|_B$

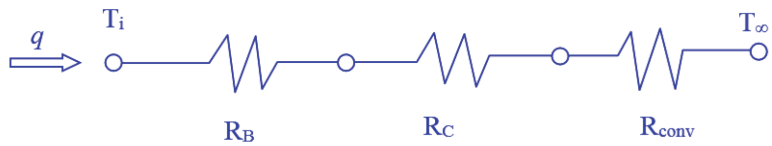
We can get $\begin{matrix} k_A < k_B \\ k_B > k_C \end{matrix}$

(c) Plot q" vs. x in the figure below. [10 pts]



Problem 1 – continued

Start your answer to part (d) here. [10 pts]



Apply energy balance: $\dot{q} \cdot A \cdot L_A = q$

$$\Rightarrow \quad q = \frac{T_i - T_\infty}{\dfrac{L_B}{k_B A} + \dfrac{L_C}{k_C A} + \dfrac{1}{hA}}$$

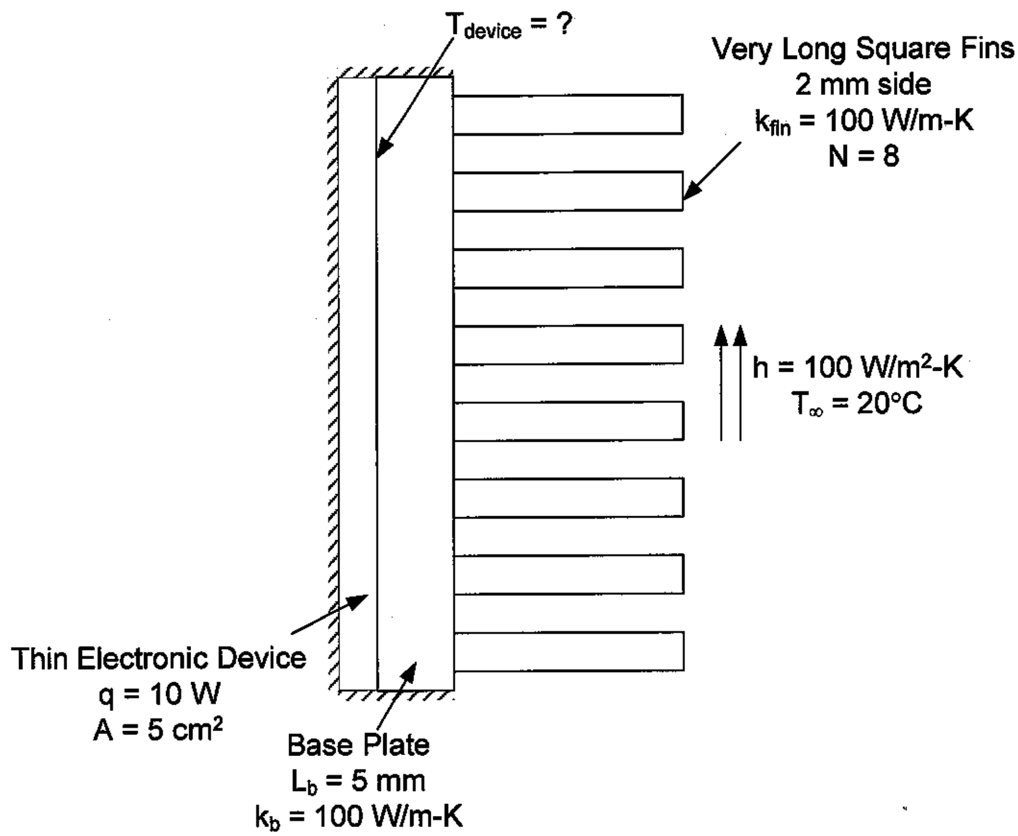
$$\Rightarrow \quad T_i = \dot{q} L_A \left(\frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h} \right) + T_\infty$$

Circle your division: 1 2 3 4 Name________________________

Problem 2 [35 points]

A thin electronic device of cross-sectional area 5 cm^2 dissipates 10 W of heat. The device is insulated on one side. A heat sink of the same surface area is directly attached to the electronic device. Thermal conductivity of the heat sink material is 100 W/(m·K). The base plate of the heat sink has a thickness of 5 mm. Eight infinitely long fins are attached to the base plate and the fins have a square cross-section of 2 mm side. Air with ambient temperature of 20°C and convective heat transfer coefficient 100 W/(m^2K) flows on the heat sink.

(a) Draw a thermal circuit for the system and label all resistances.
(b) Determine temperature (°C) at the interface between the device and the base plate.
(c) Calculate the effectiveness of a single fin.
(d) What is the value of efficiency of a single fin?



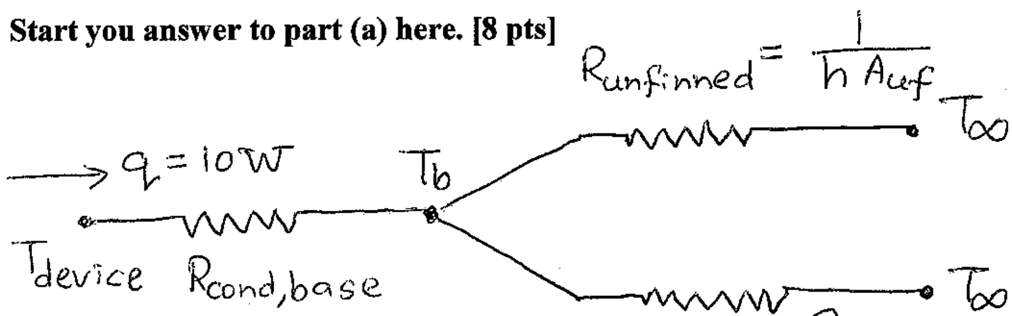
List your assumptions below. [2 pts] * Steady state

* No contact resistance between device and base plate
* No contact resistance between base plate and fins
* 1-D conduction through base plate and fins
* $h_{unfinned} = h_{fin} = h$
* Uniform convection on exposed surfaces
* Ignore radiation from exposed surfaces

Circle your division: 1 2 3 4 Name________________________

Problem 2 – continued

Start you answer to part (a) here. [8 pts]



$R_{cond,base} = \frac{L_b}{k_b A}$

$$R_{fins} = \frac{\theta_b}{8 q_{fin}} = \frac{\theta_b}{8\sqrt{hPk_{fin}A_c}\,\theta_b}$$

Start your answer to part (b) here. [15 pts]

Considering thermal. circuit:
$$q = \frac{T_{device} - T_\infty}{R_{eq.}}$$

$$R_{cond,base} = \frac{L_b}{k_b A} = \frac{5\times10^{-3}\ m}{100\ \frac{W}{m\text{-}K} \times 5\times10^{-4}\ m^2} = 0.1\ \frac{K}{W}$$

$$R_{unfinned} = \frac{1}{h A_{uf}} = \frac{1}{h(A - 8w^2)} = \frac{1}{100\ \frac{W}{m^2K}\times(5 - 8\times(0.2)^2)\times10^{-4}\ m^2}$$
$$= 21.4\ \frac{K}{W}$$

$$R_{fins} = \frac{1}{8\sqrt{h \underset{4w}{P} k_{fin} \underset{w^2}{A_c}}} = \frac{1}{8\sqrt{100\ \frac{W}{m^2K}\times 8\times10^{-3}\ m \times 100\ \frac{W}{m\text{-}K}\times(0.2)^2\times10^{-4}\ m^2}}$$
$$\cong 7\ \frac{K}{W}$$

$$R_{eq.} = R_{cond,\ base} + (R_{unfinned} \parallel R_{fins})$$
$$= 5.38\ \frac{K}{W}$$

Temperature at the interface between the device and the base plate: $T_{device} = T_\infty + q\,R_{eq.}$

$$T_{device} = 20°C + 10\ W \times 5.38\ \frac{°C}{W}$$

$$\boxed{T_{device} = 73.8°C}$$

Circle your division: 1 2 3 4 Name________________________________

Problem 2 – continued

Start your answer to part (c) here. [6 pts]

Effectiveness: $\varepsilon_{fin} = \dfrac{q_{fin}}{q_{w/o\ fin}} = \dfrac{\sqrt{hPk_{fin}A_c}\,\theta_b}{hA_c\theta_b} = \sqrt{\dfrac{Pk_{fin}}{hA_c}}$

$$\varepsilon_{fin} = \sqrt{\dfrac{8\times10^{-3}\,m \times 100\,\frac{W}{m\text{-}K}}{100\,\frac{W}{m^2K} \times (0.2)^2\times10^{-4}\,m^2}} \Rightarrow \boxed{\varepsilon_{fin} = 44.7}$$

Start your answer to part (d) here. [4 pts]

Efficiency: $\eta_{fin} = \dfrac{q_{fin}}{q_{max} = hA_{fin}\theta_b}$

$A_{fin} \to \infty$ (Infinitely long fin)

$$\Rightarrow \boxed{\eta_{fin} \to 0}$$

Problem 3 [30 points]

Part I (20 Pts): A metal alloy sphere of 2cm diameter and 20°C of uniform temperature is placed inside a convection oven at h = 20 W/(m^{2}K) and T_∞ = 800°C for thermal treatment. Neglect radiation heat exchange. *For metal alloy*: thermal conductivity = 200 W/(m·K); density = 2,000kg/m^3; specific heat = 500 J/(kg·K).

(a) Can we assume that the sphere is a lumped thermal system? Briefly justify.
(b) How long does it take (in seconds) for the surface temperature of the sphere to reach 500°C?

Part II (10 Pts): After reaching 500°C, the sphere is quenched by dipping into 100 ml of mineral oil at 20°C initial temperature. This process can be treated as convection between the sphere and the oil with h = 1,000 W/(m^{2}K) and T_∞ equal to the average oil temperature. Notice that T_∞ for oil is rising during this process. Neglect the heat loss to ambient and radiation. *For mineral oil*: density = 1,000 kg/m^3 and specific heat = 1,000 J/(kg·K). Note: 1 ml = 10^{-3} m^3

(c) Use energy conservation to find the steady state temperature (°C) of the sphere and the oil bath system.
(d) Derive a differential equation for the sphere temperature (°C) and solve to obtain an expression for sphere temperature as function of time (seconds).

Part I: Start you answer to part (a) here. [7 pts]

Assumptions: Transient, constant properties, constant h and T_∞

$$Bi_{lumped} = hL_c / k = h(r_0/3)/k = 20\times(0.01/3)/200 = 3.3\times10^{-4} \ll 0.1$$

This system is *lumped*.

Part I: Start your answer to part (b) here. [7 pts]

From equation sheet or analysis or lumped response:

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-t/\tau} \qquad \text{where } \tau = (\rho_s c_s V_s)/(hA_s) = 167\ [s]$$

Plug in numbers:

$$(500-800)/(20-800) = e^{-t/166.7} \quad \Rightarrow \boxed{t = 159\ [s]}$$

Problem 3 – continued

<u>Part II:</u> Start your answer to part (c) here. [7 pts]

From Energy conservation for the sphere/oil system:

Initial energy of sphere + initial energy of oil = Steady-state energy of both

$$\text{Constant at all time} = \rho_s c_s V_s T_s + \rho_{oil} c_{oil} V_{oil} T_{oil} = \rho_s c_s V_s T_{final} + \rho_{oil} c_{oil} V_{oil} T_{final}$$

$$\Rightarrow \quad T_{final} = \frac{\left(\rho_s c_s V_s T_s + \rho_{oil} c_{oil} V_{oil} T_{oil}\right)\Big|_{t=0}}{\rho_s c_s V_s + \rho_{oil} c_{oil} V_{oil}}$$

$$\Rightarrow \quad \boxed{T_{final} = 39.3^o C}$$

Start your answer to part (d) here. [8 pts]

From previous part:

$$\Rightarrow T_{oil} = \frac{(\rho_s c_s V_s + \rho_{oil} c_{oil} V_{oil}) T_{final} - \rho_s c_s V_s T_s}{\rho_{oil} c_{oil} V_{oil}} \Rightarrow T_{oil} = 40.95 - 0.0419 T_s$$

From energy balance:

$$\rho_s c_s V_s \frac{dT_s}{dt} = -hA_s (T_s - T_{oil}) \quad \Rightarrow \quad \frac{dT_s}{dt} = -0.3(T_s - T_{oil})$$

$$\Rightarrow \quad \boxed{\frac{dT_s}{dt} = -0.313 T_s - 39.3)}$$

Solution to this ODE

$$\frac{dT_s}{dt} = -0.313(T_s - 39.3)$$

$$\Rightarrow \frac{d\left(T_s - 39.3\right)}{dt} = -0.313(T_s - 39.3)$$

$$\Rightarrow T_s = C_1 e^{-0.313t} + 39.3$$

Use initial condition to determine C_1:

Initial condition: $T_s(t = 0) = 500\,[^o C]$

$$\Rightarrow \boxed{T_s = 460.7 \times e^{-0.313 \times t} + 39.3} \quad or \quad \boxed{T_s = 460.7 \times e^{-\frac{t}{3.19}} + 39.3}$$