ME 315
Exam 1
Thursday, October 1, 2015

- This is a closed-book, closed-notes examination. *There is a formula sheet provided. You are also allowed to bring your own one-page letter size, double-sided crib sheet.*
- You must turn off all communications devices before starting this exam, and leave them off for the entire exam.
- Please write legibly and show all work for your own benefit.
- State all assumptions.
- Please arrange all your sheets in the correct order.

Name: ____________________ ______________________
Last First

CIRCLE YOUR DIVISION

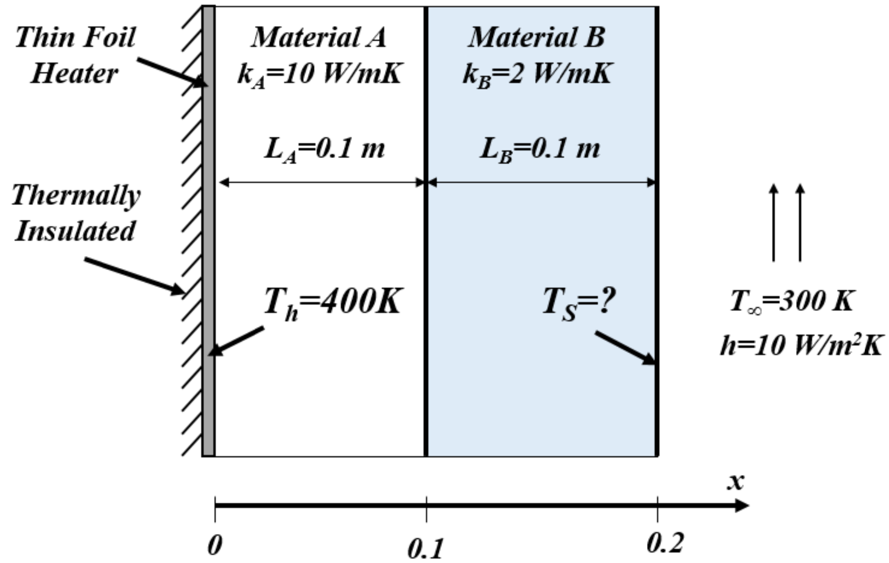
Div. 1 (8:30 am)	Div. 2 (9:30 am)	Div. 3 (11:30 am)	Div. 4 (3:30 pm)
Prof. Naik	Prof. Ruan	Prof. Pan	Prof. Marconnet

Your Assigned # : ________
(Only applicable to Div. 3)

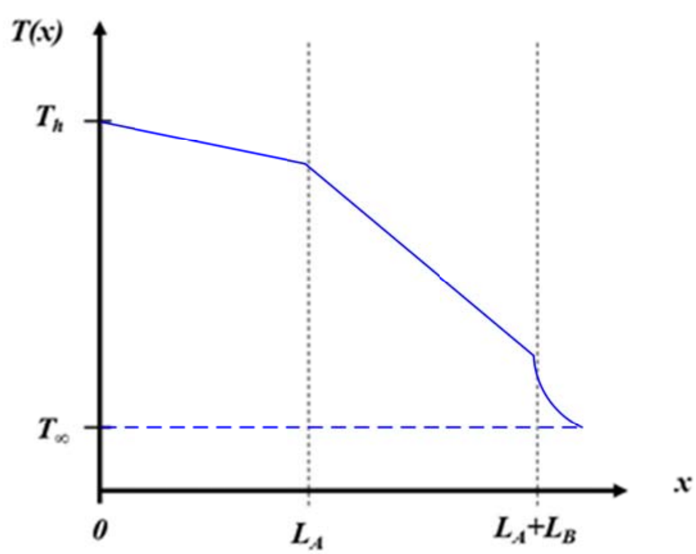
Problem	Score
1 (30 Points)	
2 (30 Points)	
3 (40 Points)	
Total (100 Points)	

Problem 1 (30 pts)

A composite plane wall is made of two slabs of materials, as shown below. A very thin foil heater is attached to the left surface of the composite wall and the heater backside is thermally insulated. The right surface of the wall is subjected to convection. The material properties and dimensions are shown as below. Assume steady state and neglect radiation and contact thermal resistances.



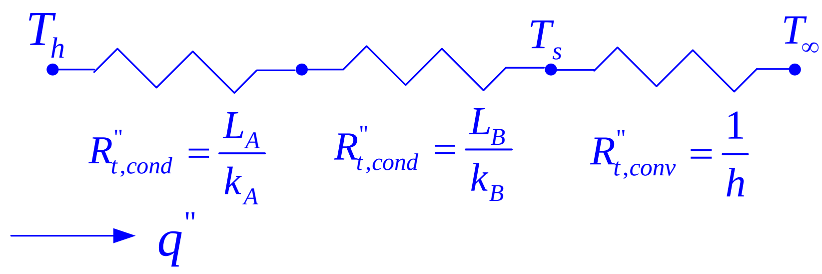
(a) (**8 pts**) On the axis below, <u>qualitatively</u> draw the temperature distribution $T(x)$ for the composite wall, from T_h to T_∞. Note that k_A is greater than k_B.



<u>Important Features:</u>

Linear profile in plane wall at steady state; slope inversely proportional to thermal conductivity

(b) (**12 pts**) Draw the thermal circuit between the heater temperature $\boldsymbol{T_h}$ and the ambient temperature $\boldsymbol{T_\infty}$. Calculate the total thermal resistance $\boldsymbol{R''_{total}}$.



$$R''_{total} = \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{1}{h} = \frac{0.1\text{ m}}{10\frac{\text{W}}{\text{m-K}}} + \frac{0.1\text{ m}}{2\frac{\text{W}}{\text{m-K}}} + \frac{1}{10\frac{\text{W}}{\text{m}^2\text{-K}}} \Rightarrow \underline{\boldsymbol{R''_{total} = 0.16\frac{\textbf{m}^2\textbf{-K}}{\textbf{W}}}}$$

(c) (**10 pts**) Find the wall surface temperature (right side) $\boldsymbol{T_S}$ and the heat flux generated by the foil heater $\boldsymbol{q''}$.

$$q'' = \frac{T_h - T_\infty}{R''_{total}} = \frac{(400-300)\text{ K}}{0.16\frac{\text{m}^2\text{-K}}{\text{W}}} \Rightarrow \underline{\boldsymbol{q'' = 625\frac{\textbf{W}}{\textbf{m}^2}}}$$

$$q'' = \frac{T_s - T_\infty}{\frac{1}{h}} \Rightarrow T_s = T_\infty + \frac{q''}{h} = 300\text{ K} + \frac{625\frac{\text{W}}{\text{m}^2}}{10\frac{\text{W}}{\text{m}^2\text{-K}}} \Rightarrow \underline{\boldsymbol{T_s = 362.5\text{ K}}}$$

Problem 2 (30 pts)

In the last few years, several companies have developed novel products to replace ice cubes. One particular brand sells spheres of granite (diameter, D = 2.5 cm; mass M = 0.025 kg) that you cool in the freezer before use.
Granite Properties: k = 1.5 W/(m K); c_p = 800 J/(kg K)

(a) (**12 pts**) In the freezer, the granite sphere is exposed to air at T_c = -10°C with a uniform convection coefficient of h_c = 5 W/(m^{2}K). Determine the time required to cool one granite sphere initially at 25°C to 0°C. Note: Characteristic length for a sphere is L_c = V/A$_s$ = D/6.

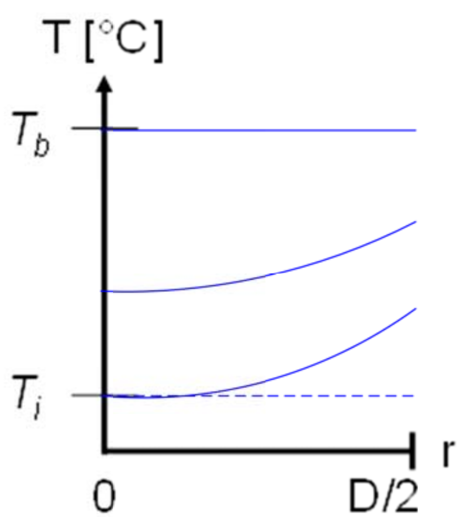
$$Bi = \frac{h_{conv}(D/6)}{k_{solid}} = \frac{5\frac{\text{W}}{\text{m}^2\text{-K}} \times \frac{2.5\times10^{-2}}{6}\text{m}}{1.5\frac{\text{W}}{\text{m-K}}} = 0.0138 \Rightarrow Bi < 0.1 \Rightarrow \text{lumped system}$$

Thermal time constant: $$\tau_t = \frac{\rho V C_p}{h_{conv}A} = \frac{MC_p}{h_{conv}\left(4\pi r_o^2\right)} = \frac{0.025\text{ kg}\times 800\frac{\text{J}}{\text{kg-K}}}{5\frac{\text{W}}{\text{m}^2\text{-K}}\times 1.96\times10^{-3}\text{ m}^2} = 2037\text{ seconds}$$

$$\frac{\theta}{\theta_i} = \frac{T-T_\infty}{T_i-T_\infty} = \exp\left(-\frac{t}{\tau_t}\right) \Rightarrow \frac{0-(-10)}{25-(-10)} = \exp\left(-\frac{t}{2037}\right) \Rightarrow \boldsymbol{t = 2552\text{ seconds}}$$

Now a single already cooled granite sphere, uniformly at T_i = 0°C, is placed in a very large warm beverage at T_b = 25°C and experiences a uniform convection coefficient of h_b = 75 W/(m^{2}K).

(b) (**6 pts**) On the axes given, qualitatively sketch the radial temperature profile at three points in time: (1) a few seconds after the cold sphere is placed in the warm beverage; (2) a few minutes after the sphere is placed in the beverage; and (3) a few hours after the sphere is placed in the beverage. No quantitative calculations are required.



Important Features:

Slope always zero at the center; slope decreases at the surface and heat penetrates more into the solid with increasing time

(c) (**6pts**) Does the heat transfer rate (q) from the beverage to the sphere increase, decrease, or stay constant throughout the process? Explain your answer with a few sentences, equations, and/or references to your sketch in (b).

$q_{conv} = h_{conv}(T_s - T_\infty) \Rightarrow$ **rate of heat from the beverage to the sphere decreases with time** because the temperature difference between the surface and warm beverage decreases while the convective heat transfer coefficient remains constant

(d) (**6 pts**) Now assume you have many granite spheres available and cooled to T_i = 0°C. Determine the final (steady state) temperature of the granite sphere-beverage system if there is 0.01 kg of the beverage per each granite sphere. The beverage has a heat capacity of c_p = 4000 J/(kg K) and the beverage is initially at T_b = 25°C. Neglect all losses from the system.

Spherical ice cubes have spatially uniform temperature initially and at steady state

Considering energy balance: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \Rightarrow 0 = \Delta E_{st} = E_f - E_i \Rightarrow E_f = E_i$

Initial energy of sphere-beverage system:

$$E_i = \left(mC_pT_i\right)_{beverage} + \left(mC_pT_i\right)_{sphere}$$

$$= 0.01\text{ kg} \times 4000\frac{\text{J}}{\text{kg-K}} \times (25+273)\text{K} + 0.025\text{ kg} \times 800\frac{\text{J}}{\text{kg-K}} \times (0+273)\text{K} = 17,380\text{ J}$$

Final energy of sphere-beverage system:

$$E_f = \left(mC_pT_f\right)_{beverage} + \left(mC_pT_f\right)_{sphere}$$

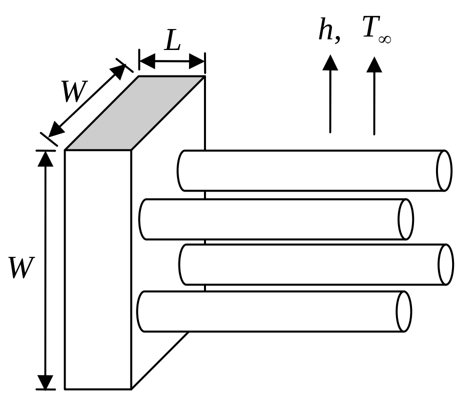
$$= 0.01\text{ kg} \times 4000\frac{\text{J}}{\text{kg-K}} \times (T_f)\text{K} + 0.025\text{ kg} \times 800\frac{\text{J}}{\text{kg-K}} \times (T_f)\text{K} = 60(T_f)\text{ J}$$

Final temperature of sphere-beverage system: $T_f = \frac{17,380}{60} \Rightarrow \boldsymbol{T_f = 289.7\text{ K}}$

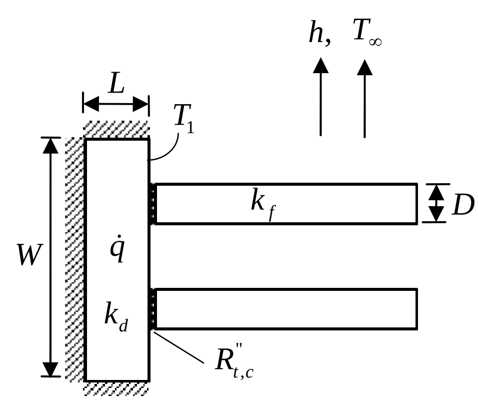
Problem 3 (40 pts)

A device has a square cross section $W \times W$ and thickness L, and its thermal conductivity is k_d. Under working conditions, the device has a uniform volumetric heat generation rate $\dot{q}$. Its right surface is exposed to the ambient air at T_∞ with a convection coefficient h, while the other five surfaces can be considered as insulated. To help cool the device, a 2 by 2 array of long pin fins (4 fins in total) with thermal conductivity k_f are bonded to the surface. The contact resistance $R''_{t,c}$ between surface of the device and the fin base resulted from the bonding process cannot be neglected. The diameter of the fins is D, and the fin length can be assumed infinitely long. The surface temperature of the device is measured (using infrared camera) to be T_1. Values of the parameters are given below.

$W = 0.1$ [m]	$L = 0.05$ [m]	$D = 0.02$ [m]	$h = 100$ [W/m^{2}K]
$T_\infty = 25$ [^{o}C]	$T_1 = 80$ [^{o}C]	$k_d = 20$ [W/mK]	$k_f = 400$ [W/mK]
$R''_{t,c} = 8\times10^{-5}$ [m^{2}K/W]			



Perspective View

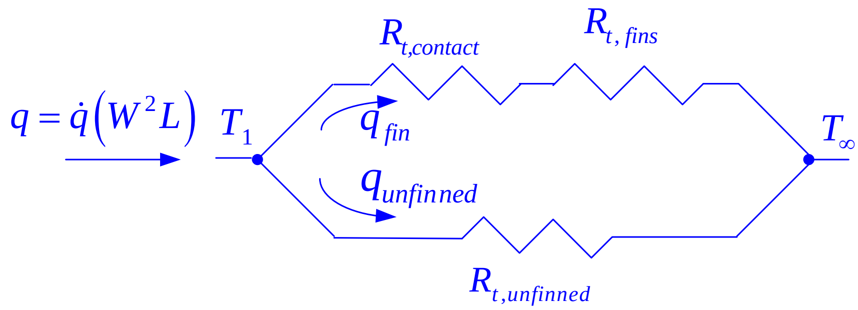


Front View

(a) **(20 pts)** Calculate the volumetric heat generation rate $\dot{q}$ in the device.

Considering energy balance for the device: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \Rightarrow \dot{q}V = \dot{q}\left(W^2L\right) = q_{fin} + q_{unfinned}$

Consider thermal circuit:



For infinitely long fin: $q_{fin} = \sqrt{hPk_f A_c}\,\theta_b$

Thermal resistance of all fins: $R_{t,\,fins} = \dfrac{\theta_b}{Nq_{fin}} = \dfrac{1}{N\sqrt{hPk_f A_c}}$; $P = \pi D$ and $A_c = \dfrac{\pi}{4} D^2$

$$R_{t,\,fins} = \frac{1}{4\sqrt{100\,\dfrac{\text{W}}{\text{m}^2\text{-K}} \times 0.0628\ \text{m} \times 400\,\dfrac{\text{W}}{\text{m-K}} \times 3.14\times10^{-4}\,\text{m}^2}} = 0.281\,\frac{\text{K}}{\text{W}}$$

Thermal contact resistance of all fins: $R_{t,contact} = \dfrac{R''_{t,c}}{NA_c} = \dfrac{8\times10^{-5}\,\dfrac{\text{m}^2\text{-K}}{\text{W}}}{4\times3.14\times10^{-4}\,\text{m}^2} = 0.064\,\dfrac{\text{K}}{\text{W}}$

Convection thermal resistance of the unfinned surface:

$$R_{t,unfinned} = \frac{1}{h\left(W^2 - NA_c\right)} = \frac{1}{100\,\dfrac{\text{W}}{\text{m}^2\text{-K}} \times \left(0.01 - 4\times3.14\times10^{-4}\right)\text{m}^2} = 1.144\,\frac{\text{K}}{\text{W}}$$

Substituting in energy balance: $\dot{q}\left(W^2 L\right) = \dfrac{T_1 - T_\infty}{R_{t,\,fins} + R_{t,contact}} + \dfrac{T_1 - T_\infty}{R_{t,unfinned}}$

$$\dot{q}\times\left(0.01\times0.05\right)\text{m}^3 = \frac{\left(80-25\right)\text{K}}{0.281\,\dfrac{\text{K}}{\text{W}} + 0.064\,\dfrac{\text{K}}{\text{W}}} + \frac{\left(80-25\right)\text{K}}{1.144\,\dfrac{\text{K}}{\text{W}}} = 159.4\ \text{W} + 48.1\ \text{W} = 207.5\ \text{W}$$

Volumetric heat generation rate in the device: $\boldsymbol{\dot{q} = 4.15\times10^5\,\dfrac{\text{W}}{\text{m}^3}}$

(b) **(10 pts)** Calculate the highest temperature in the device.

Considering heat diffusion equation in rectangular coordinates:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)+\dot{q}=\rho C_p \frac{\partial T}{\partial t} \Rightarrow \frac{d^2T}{dx^2}+\frac{\dot{q}}{k_d}=0$$

Integrate: $\frac{dT}{dx}=-\frac{\dot{q}}{k_d}x+c_1$

Integrate again: $T(x)=-\frac{\dot{q}}{2k_d}x^2+c_1x+c_2$

Boundary Conditions: $\left.\frac{dT}{dx}\right|_{x=0}=0$ and $T|_{x=L}=T_1$

$$\Rightarrow T(x)=T_1+\frac{\dot{q}L^2}{2k_d}\left(1-\frac{x^2}{L^2}\right)$$

Maximum temperature in the device:

$$T_{max}=T(x=0)=T_1+\frac{\dot{q}L^2}{2k_d}=80^\circ\text{C}+\frac{4.15\times10^5\,\frac{\text{W}}{\text{m}^3}\times(0.05)^2\,\text{m}^2}{2\times20\,\frac{\text{W}}{\text{m-K}}}\Rightarrow \underline{\boldsymbol{T_{max}=105.9^\circ\text{C}}}$$

(c) **(10 pts)** Define, **using words**, the effectiveness of a fin. Calculate the fin effectiveness of a single fin in this application. Note: Consider the contact resistance.

Fin effectiveness is the ratio of fin heat transfer rate to the rate of heat transfer without the fin from the base surface

$$\varepsilon_{fin}=\frac{q_{fin}}{hA_c\theta_b}$$

Rate of heat transfer for a single fin: $q_{fin}=\frac{(80-25)\,\text{K}}{4\times\left(0.281\frac{\text{K}}{\text{W}}+0.064\frac{\text{K}}{\text{W}}\right)}=39.2\text{ W}$

Rate of heat transfer without the fin: $q_{no,fin}=100\frac{\text{W}}{\text{m}^2\text{-K}}\times3.14\times10^{-4}\,\text{m}^2\times(80-25)\,\text{K}=1.7\text{ W}$

$$\varepsilon_{fin}=\frac{39.2\text{ W}}{1.7\text{ W}}\Rightarrow \underline{\boldsymbol{\varepsilon_{fin}=23.1}}$$