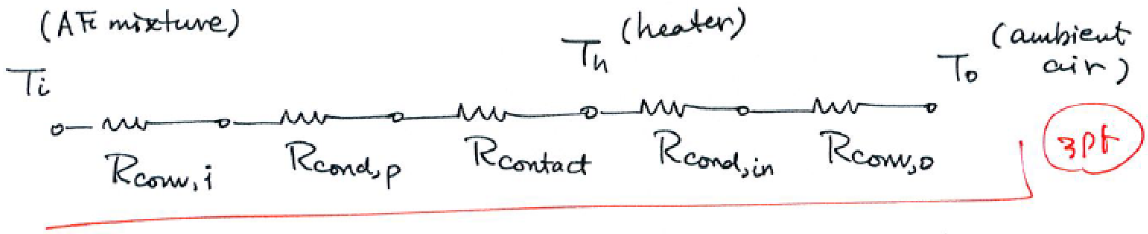
ME315 Fall 2013

Exam #1 Solutions

Sept 24, 2013

Problem 1:

1 (a)



(1pt) $R_{conv,i} = \dfrac{1}{h_i (2\pi r_1 \cdot L)}$ (Convection in the pipe)

(1pt) $R_{cond,p} = \dfrac{\ln(r_2/r_1)}{2\pi k_p \cdot L}$ (conduction of the pipe wall)

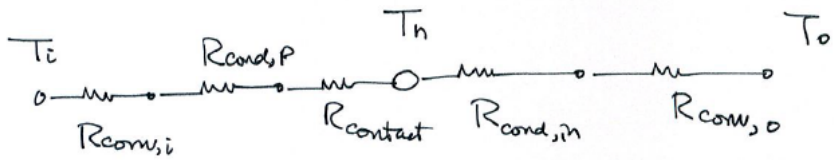
(3pt) $R_{cont} = R_c'' \cdot (2\pi r_2 L)$ (contact resistance between the heater and pipe wall)

(1pt) $R_{cond,in} = \dfrac{\ln(r_3/r_2)}{2\pi k_s \cdot L}$ (Conduction of the insulation)

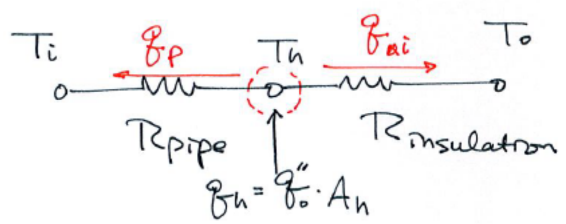
(1pt) $R_{conv,o} = \dfrac{1}{h_o (2\pi r_3 \cdot L)}$ (Convection on the outer wall of insulation).

(b)

Consider the heater node



⇓



(4pt)

$$R_{pipe} = R_{conv,i} + R_{cond,p} + R_{contact}$$

$$R_{insulation} = R_{cond,in} + R_{conv,o}$$

$$q_h = q_o'' \cdot (2\pi r_2 L)$$ heat generation by the heater. (2pt)

$$q_p = \frac{T_h - T_i}{R_{pipe}} = \frac{T_h - T_i}{\frac{1}{h_i(2\pi r_1 L)} + \frac{\ln(r_2/r_1)}{2\pi k_p L} + R_c''(2\pi r_2 L)}$$

heat flow rate from the heater to the AF mixture

(2pt)

$$q_i = \frac{T_h - T_o}{R_{insulation}} = \frac{T_h - T_o}{\frac{\ln(r_3/r_2)}{2\pi k_s L} + \frac{1}{h_o(2\pi r_3 L)}}$$

heat flow rate from the heater to the ambient flow.

(2pt)

(c) Apply the energy balance.

$$\dot{E}_{in} = \dot{E}_{out}.$$

$$\dot{E}_{in} = q_o''\,(2\pi r_2 \cdot L)$$

$$\dot{E}_{out} = q_p + q_i = \frac{T_h - T_i}{R_{pipe}} + \frac{T_h - T_o}{R_{insulation}}.$$

(5pt)

$$\therefore \quad q_o''\,(2\pi r_2 L) = \frac{T_h - T_i}{R_{pipe}} + \frac{T_h - T_o}{R_{insulation}}$$

where

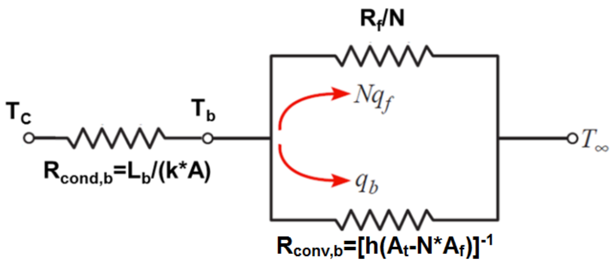
$$R_{pipe} = \frac{1}{h_i\,(2\pi r_1 L)} + \frac{\ln(r_2/r_1)}{2\pi k_p L} + R_c'' \cdot (2\pi r_2 L)$$

$$R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_s L} + \frac{1}{h_o\,(2\pi r_3 L)}$$

(5pt)

Problem 2:

(a) Draw the thermal circuit for this problem. (7 points)



Note: A_t-N*A_f = A_b = A - N*A_C. For each element and node point they identify (1pt). Correctly connect the circuit (1pt).

(b) Find the <u>conduction thermal resistance</u> and <u>convection thermal resistance</u> for heat sink baseplate, and the <u>fin thermal resistance for individual fin</u>. (3+3+6 points)

From 1D conduction assumption, we know

$R_{cond,b}=L_b/(k*A)=0.1$ K/W (3pt)

Note: identify an equation (1pt). The equation is correct (1pt). The number is correct (1pt).

$R_{conv,b} = 1/(h*A_b)=21.74$ K/W (3pt)

Note: identify an equation (1pt). The equation is correct (1pt). The number is correct (1pt).

For infinite long fin, from the expression of total fin heat loss $q_f = \sqrt{hPkA_C}\theta_b$, we get

$$R_f = \frac{1}{\sqrt{hPkA_C}} = 55.9\ K/W \quad \text{(6pt)}$$

Note: identify an equation (2pt). The equation is correct (2pt). The number is correct (2pt).

(c) Find the <u>efficiency</u> and <u>effectiveness</u> for individual fin. (3+3 points)

$$\eta_f = q_f / hA_f\theta_b = 0 \qquad \text{(3pt)}$$

Note: identify an equation (1pt). The equation is correct (1pt). The number is correct (1pt).

$$\varepsilon_f = \frac{q_f}{hA_C\theta_b} = \sqrt{hPkA_C} \Big/ \left(hA_C\right) = 44.72 \qquad \text{(3pt)}$$

Note: identify an equation (1pt). The equation is correct (1pt). The number is correct (1pt).

(d) Find the <u>overall thermal resistance</u> and <u>highest temperature</u> the device can reach during its operation. (5+5 points)

Therefore, the total thermal resistance is

$$R_{total} = R_{cond,b} + \frac{N^{-1}R_f R_{conv,b}}{N^{-1}R_f + R_{conv,b}} = 4.55\ K/W \qquad \text{(5pt)}$$

Note: setup an equation based on his/her thermal circuit drawing (2pt). The equation is correct (2pt). The number is correct (1pt).

From $q_{Max} = \frac{T_C - T_\infty}{R_{total}}$, we can get $\boxed{T_C = q_{Max}R_{total} + T_\infty = 65.5°C}$ (5pt)

Note: identify an equation (2pt). The equation is correct (2pt). The number is correct (1pt).

Problem #3:

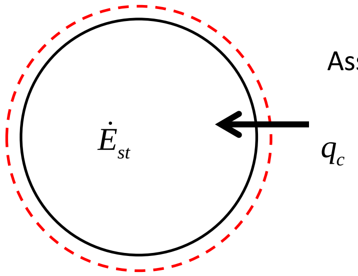
(a) (5 pts) Recall that the Biot number was derived from the ratio of the temperature gradient within the object to the temperature gradient between the object and the surroundings. Determine a new Biot number for the contact resistance case and determine if the lumped capacitance analysis is appropriate. (Hint: You may assume the critical length scale for the conduction resistance within the sphere is $L_c = R/3 = D/6$ and approximate the conductive resistance within the sphere as $R_{cond} = L_c/(k\,A_s)$, where A_s is the surface area of the sphere)

[3 pts] $$Bi_c = \frac{\Delta T_{conduction}}{\Delta T_{contact}} = \frac{qR_{cond}}{qR_{contact}} = \frac{q\left(L_c/kA_S\right)}{q\,R_c''/A_s} = \frac{L_c}{kR_c''} = \frac{D}{6kR_c''}$$

[1 pt] $$Bi_c = \frac{D}{6kR_c''} = \frac{13mm}{6\left(0.6\dfrac{W}{m\cdot K}\right)\left(0.05\dfrac{m^2K}{W}\right)} = 0.072$$

[1pt] Yes, a lumped capacitance approach is appropriate because $Bi_c < 0.1$.

(b) (15 pts) Find the differential equation that describes the temperature of the M&M and determine the chocolate temperature as a function of time. Sketch the temperature profile making sure to label the relevant temperature and time scales including the initial temperature [T_i], your hand's temperature [T_H], and the value of the temperature of the chocolate after one thermal time constant [T(t = τ) and τ] on your graph/axes.



Assumptions:

(1) Lumped capacitance valid (Bi < 0.1) → T ≠f(r)
(2) Constant properties
(3) No generation

[2 pts] Schematic and\or general energy balance: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \rightarrow \dot{E}_{in} = \dot{E}_{st}$

[1 pt] Conductive heat flux: $q_c = \frac{A_s}{R''_c}\left(T_H - T(t)\right)$

[1 pt] Change in stored energy: $\dot{E}_{st} = \rho C_p V \frac{dT}{dt}$

[1 pt] Differential Equation: $\frac{A_s}{R''_c}\left(T_H - T(t)\right) = \rho C_p V \frac{dT}{dt}$

They may list each term in a single step from the energy balance for all 3 pts if correct.

or $-\frac{A_s}{R''_c}\theta = \rho C_p V \frac{d\theta}{dt}$, where

$\theta = T - T_H$

[2 pts] $-\frac{A_s}{R''_c}\theta = \rho C_p V \frac{d\theta}{dt}$ → $-\frac{A_s}{\rho C_p V R''_c} dt = \frac{1}{\theta} d\theta$

→ $-\frac{A_s}{\rho C_p V R''_c} t = \ln\theta + C_1$ →

$$\theta = \theta_i \exp\left(-\frac{A_s}{\rho C_p V R''_c} t\right) = \theta_i \exp\left(-\frac{t}{\tau}\right)$$

(Derivation or recognizing the form from lumped capacitance model are acceptable)

[2 pt] Time Constant:

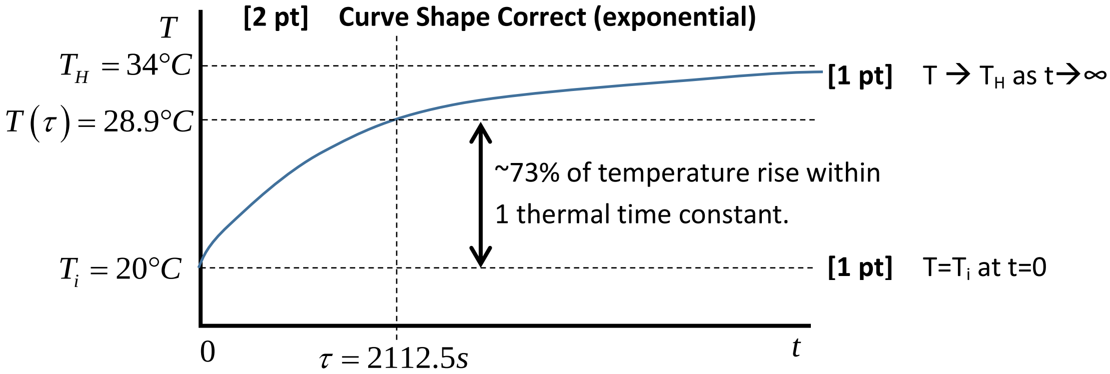
$$\tau = \left(\frac{A_s}{\rho C_p V R''_c}\right)^{-1} = \left(\frac{4\pi r^2}{\rho C_p \frac{4}{3}\pi r^3 R''_c}\right)^{-1} = \frac{\rho C_p D R''_c}{6}$$

$$= \frac{\left(1300\, {}^{kg}/_{m^3}\right)\left(1500\, {}^{J}/_{kg\cdot K}\right)(13mm)\left(0.05\, {}^{m^2K}/_{W}\right)}{6} = \boxed{211.25s}$$

[2 pt]

$$\frac{\theta(t=\tau)}{\theta_i} = \exp(-1) = 0.367$$

$$T(t=\tau) = 34°C + (20°C - 34°C)(0.367) = \boxed{28.9°C}$$



(c) (5 pts) How long you would need to hold the M&M before it began to melt?

[3 pts] $\theta_m = T_m - T_H = \theta_i \exp\left(-\frac{t_m}{\tau}\right) \rightarrow \quad t_m = -\tau \log\left(\frac{T_m - T_H}{T_i - T_H}\right)$

[2 pts] $t_m = -(211.25)\log\left(\frac{30°C - 34°C}{20°C - 34°C}\right) = 264.6s$

(d) (10 pts) Assuming that the temperature inside your mouth is also T_h = 34°C, explain why might the M&M begin to melt in your mouth within about 10 seconds? Calculate the Biot number for this case and determine if the lumped capacitance approach still valid for this condition?

[5 pts] Thermal time constant is significantly reduced (τ~10s). Considering the parameters in τ, the only value that could likely change is R''_c. Thus, decreased contact resistance could explain reduced time constant.

[1 pts] Assuming that lumped capacitance model still applies, calculate the new boundary resistance:

$$\tau = \frac{\rho C_p D R''_c}{6} \rightarrow R''_c = \frac{6\tau}{\rho C_p D} = \frac{6(10s)}{\left(1300 \, {}^{kg}\!/\!_{m^3}\right)\left(1500 \, {}^{J}\!/\!_{kg \cdot K}\right)(13mm)} = 0.0024 \frac{m^2 K}{W}$$

[1 pts] Calculate the new Biot number: $Bi_c = \frac{D}{6R''k} = \frac{(13mm)}{6\left(0.0024 \frac{m^2 K}{W}\right)\left(0.6 \, {}^{W}\!/\!_{m \cdot K}\right)} = 15$

[3 pt] No, the lumped capacitance is no longer valid since Bi > 0.1.

Some students may intuitively get to the answer that lumped capacitance approach is not valid without strictly calculating the new Bi number and R" and could receive the 3 pts for their answer (if justified logically), but not the 2 pts for calculating R" and Bi_c.

Note, that the calculated new R" is not necessarily valid because equation for time constant was derived from lumped capacitance approach.