Name:___________________________________
Last **First**

CIRCLE YOUR DIVISION:

Div. 1 (9:30 am)	**Div. 2 (11:30 am)**	**Div. 3 (2:30 pm)**
Prof. Ruan	**Prof. Naik**	**Mr. Singh**

School of Mechanical Engineering
Purdue University
ME315 Heat and Mass Transfer

Exam #1

Wednesday, September 22, 2010

Instructions:

- Write your name on each page
- Closed-book exam – a list of equations is given
- Please write legibly and show all work for your own benefit. Write on one side of the page only.
- Keep all pages in order
- <u>You are asked to write your assumptions and answers to sub-problems in designated areas. Only the work in its designated area will be graded.</u>

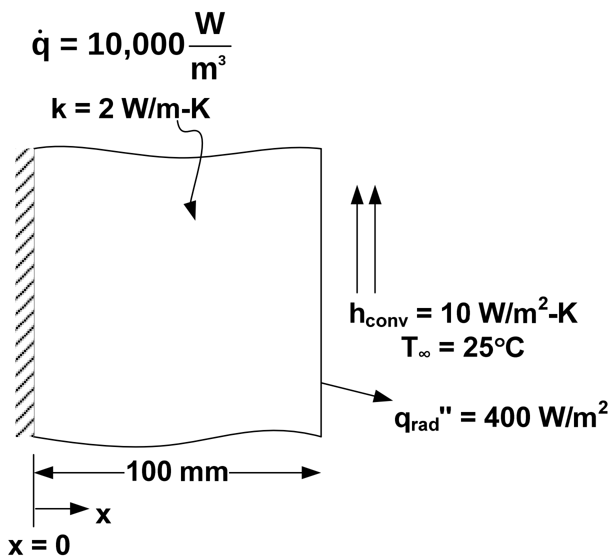
Performance

1	**30**	
2	**35**	
3	**35**	
Total	**100**	

Name:________________________________
Last First

Problem 1 [30 pts]

Consider a plane wall of thickness 100 mm and thermal conductivity 2 W/m-K shown below. One side of the wall is perfectly insulated. The other side of the wall is exposed to surrounding air at temperature 25°C and convective heat transfer coefficient of 10 W/m^2-K. The radiative heat loss from the same side to surrounding air is 400 W/m^2. There is uniform volumetric heat generation of 10,000 W/m^3 in the wall.



Assume that conduction through the wall is one-dimensional and at steady-state.

(a) Calculate the temperature (°C) of the wall surface exposed to the surrounding air.

(b) Write the differential equation and the necessary boundary conditions to obtain temperature distribution T(x) through the wall. Start with the generalized heat diffusion equation in rectangular coordinate system. Solve the differential equation to obtain T(x).

List your assumptions here [3 pts]:

Steady-state
One-dimensional conduction in the wall
Constant Properties
Uniform convective heat transfer coefficient on the surface

Start your answer to part (a) here [10 pts]:

Consider energy balance at the surface: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \Rightarrow \dot{q}V = q_{conv} + q_{rad}$

$$\Rightarrow \dot{q}(L \times A) = hA(T_s - T_\infty) + q''_{rad}A \Rightarrow 10{,}000\frac{\text{W}}{\text{m}^3} \times \frac{100}{1000}\text{m} = 10\frac{\text{W}}{\text{m}^2\text{-}^\circ\text{C}} \times (T_s - 25)^\circ\text{C} + 400\frac{\text{W}}{\text{m}^2}$$

$\underline{\boldsymbol{T_s = 85^\circ \text{C}}}$

Name:________________________________
Last First

Problem 1 - cont.

Start your answer to part (b) here [17 pts]:

Consider heat diffusion equation in rectangular coordinates:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)+\dot{q}=\rho C_p \frac{\partial T}{\partial t} \Rightarrow \underline{\boldsymbol{\frac{d^2T}{dx^2}+\frac{\dot{q}}{k}=0}}$$

Boundary Conditions: (1) $\boldsymbol{\frac{dT}{dx}=0}$ **at** $\boldsymbol{x=0}$ (insulated); (2) $\boldsymbol{T=T_s}$ **at** $\boldsymbol{x = 100}$ **mm**

Integrating: $\frac{dT}{dx}=-\frac{\dot{q}x}{k}+c_1$

Using (1), we get: $c_1=0 \Rightarrow \frac{dT}{dx}=-\frac{\dot{q}x}{k}$

Integrating again: $T(x)=-\frac{\dot{q}x^2}{2k}+c_2$

Using (2), we get: $T_s=85^\circ\text{C}=-\frac{10,000\frac{\text{W}}{\text{m}^3}\times\left(\frac{100}{1000}\right)^2\text{m}^2}{2\times 2\frac{\text{W}}{\text{m-K}}}+c_2 \Rightarrow c_2=110^\circ\text{C}$

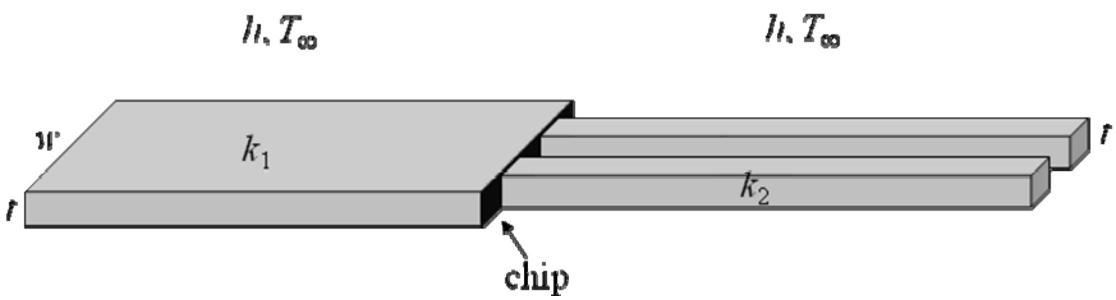
Temperature distribution: $\underline{\boldsymbol{T(x)=-\frac{\dot{q}x^2}{2k}+110^\circ\text{C}}}$

Name:________________________________
Last First

Problem 2 [35 pts]

An electronic chip has a rectangular cross section (width w = 80 mm, and height t = 20 mm) and negligible thickness, as shown in the figure. It dissipates heat at the rate of q = 20 W. To assist heat removal to the ambient, a **long** fin made of aluminum with the same cross section and with a thermal conductivity k_1 = 237 W/m-K is mounted to the left side of the chip. On the right side, two **long** fins made of copper, with a square cross section 20 mm × 20 mm are mounted, and the thermal conductivity is k_2 = 400 W/m-K. The system is exposed to the ambient air with a convective heat transfer coefficient h = 100 W/m^2-K and far-field temperature T_∞ = 24 °C.

(a) Find the steady-state temperature (°C) of the chip T_c.
(b) **Using words,** define the concept "fin effectiveness". Then evaluate the effectiveness of the aluminum fin starting from your definition.



List your assumptions here [3 pts]:
Steady-state
One-dimensional conduction in fins
Constant Properties
Negligible contact resistance
Convective heat transfer coefficient same for fin and unfinned surface

Start you answer to part (a) here [22 pts]:
Consider energy balance for the chip: $q = q_{fin,Al} + 2q_{fin,Cu} + q_{unfinned,Cu}$

$$q_{fin,Al} = M = \sqrt{hP_{Al}k_1A_{c,Al}}\,\theta_b\,;\ P_{Al} = 2(w+t) \text{ and } A_{c,Al} = wt$$

$$q_{fin,Cu} = M = \sqrt{hP_{Cu}k_2A_{c,Cu}}\,\theta_b\,;\ P_{Al} = 4t \text{ and } A_{c,Al} = t^2$$

$$q_{unfinned,Cu} = hA_{unfinned}\theta_b\,;\ A_{unfinned} = \left(wt - 2t^2\right)$$

$$\Rightarrow 30\ \text{W} = \sqrt{100\frac{\text{W}}{\text{m}^2\text{-K}}\times 2\left(\frac{80}{1000}+\frac{20}{1000}\right)\text{m}\times 237\frac{\text{W}}{\text{m-K}}\times\left(\frac{80}{1000}\times\frac{20}{1000}\right)\text{m}^2}\,(T_c - T_\infty)$$

$$+2\sqrt{100\frac{\text{W}}{\text{m}^2\text{-K}}\times 4\left(\frac{20}{1000}\right)\text{m}\times 400\frac{\text{W}}{\text{m-K}}\times\left(\frac{20}{1000}\right)^2\text{m}^2}\,(T_c - T_\infty)$$

$$+100\frac{\text{W}}{\text{m}^2\text{-K}}\times\left[\left(\frac{80}{1000}\times\frac{20}{1000}\right) - 2\left(\frac{20}{1000}\right)^2\right]\text{m}^2\,(T_c - T_\infty)$$

$\underline{\mathbf{T_c = 31^\circ C}}$

Name:____________________________________

Last First

Problem 2 - cont.

Start your answer to part (b) here [10 pts]:

Fin effectiveness is the ratio of the rate of heat transfer with the fin to the rate of heat transfer that would exist without the fin from the cross-sectional area of the fin at the base.

$$\varepsilon_{fin,Al} = \frac{q_{fin,Al}}{hA_c\theta_b} = \frac{M}{hA_c\theta_b} = \frac{\sqrt{hP_{Al}k_1A_{c,Al}}\theta_b}{hA_{c,Al}\theta_b} = \sqrt{\frac{k_1P_{Al}}{hA_{c,Al}}} = \sqrt{\frac{237\dfrac{\text{W}}{\text{m-K}}\times 2\left(\dfrac{80}{1000}+\dfrac{20}{1000}\right)\text{m}}{100\dfrac{\text{W}}{\text{m}^2\text{-K}}\times\left(\dfrac{80}{1000}\times\dfrac{20}{1000}\right)\text{m}^2}}$$

$$\boldsymbol{\varepsilon_{fin,Al} = 17.2}$$

Name:____________________________________

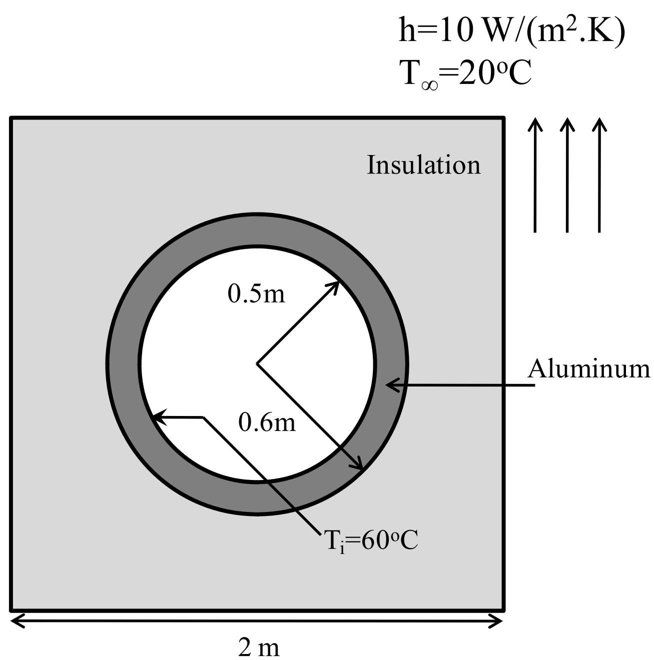
Last **First**

3. Problem 3 [35 pts]

A cylindrical aluminum pipe with inner radius $r_1 = 0.5$ m and outer radius $r_2 = 0.6$ m as shown below carries water. The inner surface of the pipe is at a temperature $T_i = 60^oC$. Square insulation of width 2 m experiences uniform convection heat transfer at $h = 10$ W/m^2-K and $T_\infty = 20^oC$. Assume the following: $k_{aluminum} = 230$ W/m-K, $k_{insulation} = 2$ W/m-K, and unit length of the pipe.

(a) Assuming that the heat conduction through the cylindrical aluminum pipe is one-dimensional in the radial direction, draw the thermal resistance network from T_i to T_∞. Clearly label all the temperatures and calculate all the resistances involved (using appropriate 1D resistances and 2D shape factors).

(b) Calculate the heat transfer rate (W) from the inner surface to the ambient.



List your assumptions here [3 pts]:

Stead- state

Constant properties

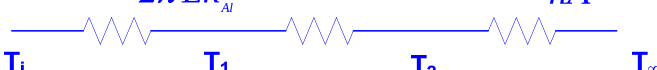
One-dimensional conduction through the aluminum pipe; two-dimensional conduction through the insulation

Uniform convective heat transfer coefficient on the insulation surface

Start you answer to part (a) here [25 pts]:

Thermal resistance network is shown below.

$$R_{cond,1D} = \frac{\ln\left(r_2/r_1\right)}{2\pi L k_{Al}} \qquad R_{conv} = \frac{1}{hA}$$



$$R_{cond,2D} = \frac{1}{S k_{insulation}}$$

Name:______________________________

Last **First**

Problem 3 - cont.

1D conduction resistance of the cylindrical aluminum pipe:

$$R_{cond,1D} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k_{Al}} = \frac{\ln\left(\frac{0.6\text{ m}}{0.5\text{ m}}\right)}{2\pi \times 1\text{ m} \times 237\frac{\text{W}}{\text{m-K}}} \Rightarrow \boldsymbol{R_{cond,1D} = 0.0001\frac{K}{W}}$$

2D conduction resistance of the insulation:

$$R_{cond,2D} = \frac{1}{Sk_{insulation}}\text{; Using Table 4.1: } S = \frac{2\pi L}{\ln\left(\frac{1.08w}{D=2r_2}\right)} = \frac{2\pi \times 1\text{ m}}{\ln\left(\frac{1.08\times 2\text{ m}}{1.2\text{ m}}\right)} = 10.7\text{ m}$$

$$R_{cond,2D} = \frac{1}{10.7\text{ m} \times 2\frac{\text{W}}{\text{m-K}}} \Rightarrow \boldsymbol{R_{cond,2D} = 0.0467\frac{K}{W}}$$

Convection resistance on the insulation surface:

$$R_{conv} = \frac{1}{hA} = \frac{1}{h\times 4wL} = \frac{1}{10\frac{\text{W}}{\text{m}^2\text{-K}}\times(4\times 2\times 1)\text{m}^2} \Rightarrow \boldsymbol{R_{conv} = 0.0125\frac{K}{W}}$$

Start your answer to question (b) here [7 pts]:

Total resistance: $R_{total} = R_{cond,1D} + R_{cond,2D} + R_{conv} = 0.0593\frac{\text{K}}{\text{W}}$

Heat transfer rate from the inner surface to the ambient: $q = \frac{T_i - T_\infty}{R_{total}} = \frac{(60-20)\text{K}}{0.0593\frac{\text{K}}{\text{W}}}$

$\boldsymbol{q = 674.5\text{ W}}$